2016

MATHEMATICS - I

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be written either in English or in Bengali but all answers must be in one and the same language.

1. Answer any two questions :-

(a) Consider the following subspaces of IR^5 :

$U = \text{span}\{(1,3,-2,2,3), (1,4,-3,4,2), (2,3,-1,-2,9)\}$

$W = \text{span}\{(1,3,0,2,1), (1,5,-6,6,3), (2,5,3,2,1)\}$

Find a basis and the dimension of (a) $U + W$ and (b) $U \cap W$

(b) Let $f : IR^4 \longrightarrow IR^3$ be the linear mapping defined by

$f(x,y,z,w) = (x-y+z+w,\ x+2z-w,\ x+y+3z-3w)$

Find a basis and the dimension of the (i) image of f and (ii) kernel of f.

(c) Find all eigen values and corresponding eigen vectors of

$A = \begin{pmatrix} 3 & -4 \\ 2 & -6 \end{pmatrix}$

10 x 2

2. Answer any two questions :-

(a) Use Cauchy's criterion to prove that the sequence $\{x_n\}$ where

$x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!}$ is convergent.

(b) Evaluate $\lim_{x \to 0} (1 + \frac{1}{x})^x$

(c) If $y = \frac{1}{ax + b}$, find y_n

10 x 2

3. Answer any two questions :-

(a) Show that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$ where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$

(b) Show that the maximum value of $(\frac{1}{x})^x$ is $e^{\frac{1}{e}}$

(c) Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

10 x 2

4. Answer any two questions :-

(a) Let $f_n(x) = \frac{n + \cos x}{2n + \sin^2 x}$, $x \in IR$, $n \in N$. Show that $\{f_n\}_n$ converges uniformly on IR.

(b) Let $f(x) = \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show that f is Riemann integrable in $[-1, 1]$.

(c) Let p be a positive integer. Determine (as p varies) the radius of convergence of $\sum_{k=0}^{\infty} \frac{(k!)^p}{(pk)!}$ for all k.

10 x 2

5. Answer any two questions :-

(a) If $ax^2+2hxy+by^2+2gx+2fy+c = 0$ represents a pair of parallel straight lines, show that the distance between them is $2\sqrt{\frac{g^2-ca}{a(a+b)}}$

(b) Show that the condition that the line $\frac{1}{r} = a\cos\theta + b\sin\theta$ may touch the conic $\frac{l}{r} = 1 + e\cos\theta$ is $(la-e)^2 + b^2l^2 = 1$.

(c) If the normal to an ellipse at the point P meets the major and minor axes at G and H respectively, prove that PG.PH=SP.S'P where S, S′ are foci of the ellipse. 10 x 2

6. Answer any two questions :-

(a) Find the equation of the plane which passes through the line of intersection of the planes $x-2y+3z+4=0$ and $2x-3y+4z-1 = 0$ and is perpendicular to the plane $3x-y+2z-1 = 0$

(b) Show that the surface generated by the straight line which intersects the lines $x+y=0$, $z=0$; $x-y=z$, $x+y=2a$ and the parabola $x^2 = 2az$, $y = 0$ is $x^2-y^2 = 2az$.

(c) A plane passing through a fixed point (a,b,c) cuts the axes in A,B,C. Show that the locus of the centre of the sphere OABC is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 2$$

10 x 2

7. Answer any two questions :-

(a) Obtain the complete primitive and the singular solution of the equation $y = px + \sqrt{1 + p^2}$

(b) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$

(c) Solve the equation, by the method of variation of parameters

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

10 x 2

8. Answer any two questions :-

(a) Solve : $z^2 - pz + qz + (x+y)^2 = 0$

(b) Using Charpit's method, find the complete integral of the equation $2(z + xp + yq) = yp^2$

(c) Solve : $(D^2 + m^2)y = a\cos nt$, given $y = Dy = 0$, when $t = 0$, $D = \frac{d}{dt}$

10 x 2

9. Answer any two questions :-

(a) Two uniform similar rods PQ and QT of same material of length 2a and 2b respectively are rigidly united at Q and suspended freely from P. If they rest inclined at angles α and β respectfully to the vertical, prove that -

$$(a^2 + 2ab)\sin\alpha = b^2\sin\beta$$

Contd...P/3.

(b) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and the plane face is horizontal by a weight P attached at a point in the rim. Prove that the co-efficient of friction is

$$\frac{P}{\sqrt{W(2P + W)}}$$

(c) Four equal rods each of weight W form a rhombus ABCD with smooth hinges at the joints. The frame is suspended by the end A and a weight W′ is attached at C. A stiffering rod of negligible weight joins the middle points of AB, AD keeping these inclined at an angle α to AC. Show that the thrust in the stiffering rod is $(4W + 2W')\tan\alpha$

10 x 2

10. Answer <u>any two</u> questions :-

(a) The velocities of a point parallel to the axes x and y are $u + wy$ and $v + w'x$ respectively, where u, v, w, w′ are constants. Show that the path of the point is a conic.

(b) A particle moving with a simple harmonic motion in a straight line has velocities v_1, v_2 at distances x_1, x_2 from the centre of its path. Show that, if T be the period of its motion, then

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(c) Assuming the moon to describe a circular orbit of radius 4×10^5 km round the earth in 27.3 days, calculate the periodic time of an artificial satellite of the earth near the earth's surface (Radius of the earth = 6400 km). 10 x 2

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