2016

# MATHEMATICS PAPER-II

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be written either in English or in Bengali but all answers must be in one and the same language.

## GROUP - A

Answer any five questions

1. (a) If $\text{Cosh}^{-1}(x + iy) + \text{Cosh}^{-1}(x - iy) = \text{Cosh}^{-1} a$ where $x, y, a$ are real and $a > 1$ prove that $(x, y)$ lies on an ellipse.

   (b) Using ${}^{n}C_{r}$ is a positive integer for $n, r \in \mathbb{N}$ prove that product of r consecutive natural numbers is divisible by $\lfloor r$ 14 + 14

2. (a) Solve $2x^4 + x^3 + 2x^2 + 3x + 18 = 0$ where product of two roots is equal to product of other two. 14

   (b) If a, b, c are sides of a triangle prove that -

   $$a^3+b^3+c^3 < a^2(b+c) + b^2(c+a) + c^2(a+b) < 2(a^3+b^3+c^3)$$ 5 + 9

3. (a) Let $\{G, +\}$ be an abelian group with identity element denoted by 0. Define with proper justification a binary operation '.' such that $\{G, +, \cdot\}$ is a commutative ring. 14

   (b) If G be a group of order $p^2$ where p is prime then prove that G is abelian. 14

4. (a) Use Lagrange method to find the points on the parabola $x^2 = 4y$ which are nearest to (0, 1). 14

   (b) A ball follows the path $\Gamma : [-1, 1] \longrightarrow \mathbb{R}^3$ defined by $\Gamma(t) = (|t|, |t - \frac{1}{2}|, t)$. Find the distance travelled by the ball. 14

5. (a) Evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$ over the boundary surface S of the region R above xy plane bounded by the core $x^2 + y^2 = z^2$ and $z = 4$ where $\vec{F} = 4xz\hat{i} + xyz^2\hat{j} + 3z\hat{k}$ 14

   (b) Prove that $f(x, y) = \sqrt{|xy|}$ is continuous at (0, 0) but is not differentiable at (0, 0). 5 + 9

6. (a) Prove that $\left[\vec{l}\ \vec{m}\ \vec{n}\right]\left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$ 14

   (b) Let S denote the unit circle $\{(\cos\theta, \sin\theta) ; 0 \le \theta \le 2\pi\}$ prove that $d(x_1, x_2) = |\theta_1 - \theta_2|$ is a metric on S where $x_1 = (\cos\theta_1, \sin\theta_1)$ and $x_2 = (\cos\theta_2, \sin\theta_2)$. What are the open sets in S ? 6 + 8

7. (a) If Lagrange interpolation formula is expressed as

$$Ln(x) = \sum_{i=0}^{n} w_i(x) y_i$$ which interpolate f(x) where $y_i = f(x_i)$ for i = 0(1)n, then write expression of $w_i(x)$. Also state and prove properties of $w_i(x)$. 2 + 3 + 9

(b) From Newton Coter quadrature formula prove Simpson's 1/3rd rule. Also find its composite form. 10 + 4

8. (a) Write an algorithm or draw a flow chart or write a program in C to convert a positive integer $(N)_{10}$ into octal. 14

(b) A Committee of 3 persons A, B, C decides proposals by a majority of vote. A has a voting weight 3, B has a voting weight 2 and C has a voting weight 1. Design a switching circuit so that light will glow when a majority of votes is cast in favour of the proposal. 14

## GROUP - B

Answer any two questions

9. (a) From the numbers 1, 2, 3 ....., (2n+1) three are chosen at random. Find the probability that these are in A.P. 12

(b) A continuous random variable X has the following p.d.f

$f(x) = x \quad 0 < x < 1$

$= K - x \quad 1 < x < 2$

Find K. Calculate $P(\frac{1}{2} < x \leq \frac{3}{2})$. Find mean of X. 6 + 6 + 6

10. (a) If x, y are two random variable connected by 4x + 7y = 2, find correlation co-efficient of x, y.

(b) Estimate x when y = 18 by the method of least squares :-

| x | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| y | 12 | 15 | 17 | 22 | 24 | 30 |

11. (a) Prove analytically that in $E^2$ the set $x = \{(x,y) ; y^2 \leq 4x\}$ is a convex set but $y = \{(x,y) ; y^2 \geq 4x\}$ is not a convex set. 6+6

(b) A firm manufacturing a single product has three plant I, II, III. The three plants have produced 60, 35, 40 units respectively during the month. They had made a commitment to sell 20 units to customer A, 30 units to customer B, 20 units to customer C, 30 units to customer D. Find the minimum cost of shifting the manufactured product to the five customers. The cost matrix is given below :-

| | A | B | C | D |
|---|---|---|---|---|
| I | 4 | 1 | 3 | 4 |
| II | 2 | 3 | 2 | 2 |
| III | 3 | 5 | 2 | 4 |

Is this solution unique ? why ? 14 + 4