2016

PHYSICS - PAPER-I

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali but all answers must be in one and the same language.

Group A

Answer any three questions

1. a) The plane polar coordinates of a particle moving on a plane are given by $r=a\sin\omega_1 t$ and $\theta=\omega_2 t$. Obtain expression for polar components of velocity and acceleration of the particle. 8

b)Show from Lagrange's equation of motion that the generalized momentum is conserved if the corresponding generalized coordinate is cyclic. 6

c)A Lagrangian is given in the form $L = \frac{1}{2}\alpha\left(\frac{\partial q}{\partial t}\right)^2 - \frac{1}{2}\beta q^2$ where α and β are constants. Obtain the Lagrange's equation of motion and hence find the Hamiltonian of the system. 6

d) A body of mass m hangs from one end of a light rope which is wound on the surface of a horizontal cylinder of radius 'a', which is free to rotate about its axis. Prove that the body descends with an acceleration $f=g/[1+(I/ma^2)]$ where I is the moment of inertia of the cylinder about its axis. 10

e)Calculate the gravitational potential due to a circular plate of radius 'a' , mass m at an axial point. 10

2. a) Show that a shear is equivalent to a compression and an equal extension at right angles to each other. 8

b) A wire of diameter 1 mm is kept stretched. If the temperature drops by 20 °C, how much additional weight is to be applied to keep the length constant.? (Given Young's modulus $Y=20\times10^{10}$ N/m^2 and the coefficient of linear expansion is $\alpha= 1\times10^{-5}$ /°C.) 8

c)Explain the terms surface tension and surface energy and obtain the relation between them. 6 + 8

d) Calculate the loss of energy if 1000 drops of water each 2 mm diameter coalesce to form one large drop. (Surface tension of water = 72 dynes/cm). 6

e) State and explain Bernouilli's theorem. 4

3. a)Explain the concept of length contraction and time dilation in relativity. 8

b)Two events occur, separated by a time interval of 4 seconds, at the same place in a certain inertial frame S', moving at a constant velocity 'v' , w.r.t. another inertial frame

S, the relative motion being along x-x' direction. What is the spatial separation between these two events in frame S, if the events are separated by a time interval of 6 seconds ?
6

c)Show that for a relativistic particle, the kinetic energy is $K = m_0c^2 [\frac{1}{\sqrt{1-v^2/c^2}} - 1]$ Where m_0 is the rest mass of the particle, v is the velocity of the particle and c is the speed of light. 8

d) A battery connected to a flashlight bulb is exactly counterpoised on the pan of a balance. The battery maintains an average current of 0.1 A at an average voltage of 1 Volt for 3 hours. Assuming all this energy is radiated away, what must be the order of magnitude of the sensitivity of the balance if a deflection is to be detected ? 6

e)State Fourier's theorem regarding the expansion of a periodic function. Expand the following function into a Fourier series.

$y = a$ from $t = 0$ to $t = T/2$
$y = 0$ from $t = T/2$ to $t = T$ 4+ 8

4. a)A simple harmonic wave is represented by $y = 10 \sin(\frac{2\pi t}{T} + \alpha)$. At t=0, the displacement is 5 cm, the time period being 30 second. Calculate the phase angle at t=7.5 sec and the phase difference between two positions at a time interval of 6 secs. 6

b) Explain analytically the formation of a wave packet. 6

c) The phase velocity of a transverse wave of wavelength λ in a crystal is given by

$$v = v_0 \frac{\sin(\pi a/\lambda)}{(\frac{\pi a}{\lambda})}$$

where 'a' is the average interatomic separation and v_0 is a constant. Show that the ratio of phase velocity and group velocity is given by

$$v/v_g = v_0 \frac{\tan(\pi a/\lambda)}{(\frac{\pi a}{\lambda})}$$ 8

d) Obtain the system matrix for refraction at a curved surface. 6

e)Explain the formation of interference pattern in thin films. Obtain analytically the condition for interference maxima and minima for both reflected and transmitted light. What can you conclude regarding the pattern produced by the reflected and transmitted light ? 12+2

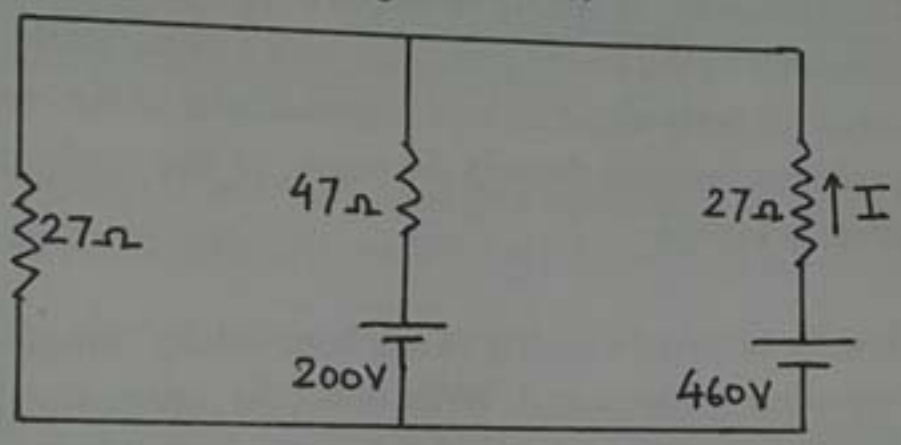
5. a)State Gauss's law in electrostatics. Hence calculate the electric field at a point inside a uniformly charged solid sphere. 4+6

Contd...P/3.

b) Use Laplace's equation to calculate the electric potential at a point in between two coaxial hollow cylinders of radii a and b (a> b), the inner cylinder is grounded and the outer one is at a potential V_a. 8

c)Show that the time average Poynting vector for a time-varying electromagnetic field is given by $\langle\vec{S}\rangle = \langle\vec{E} \times \vec{H}\rangle = \frac{1}{2}$ Real $(\vec{E} \times \vec{H}^*)$ where $\vec{E}$ and $\vec{H}$ are electric and magnetic field vectors. 6

d)Find the current I in the network given below.



6

e)Obtain the expression for instantaneous current in a series L-C-R circuit and hence explain why it is called an acceptor circuit ? 10

6. a)Apply the first law of thermodynamics to show that the difference of molar specific heats is given by

$$C_p - C_v = [p + (\frac{\partial U}{\partial V})_T] (\frac{\partial V}{\partial T})_P$$

Where U, P, V, T are internal energy, pressure, volume & temperature respectively. 8

b) Briefly describe the operation of a Carnot cycle with the help of a P-V diagram and calculate its efficiency.
Two Carnot engines A and B are operated in series. The first one A, receives heat at 900K, and rejects heat to a reservoir at temperature T K. The second engine B, receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 400 K. Calculate the temperature T for the situation when (i) the work outputs of the two engines are equal, (ii) the efficiencies of the two engines are equal. 10+8

c)Explain what is meant by Joule-Thompson effect. 6

d) Calculate the amount of heat transferred during an isothermal expansion of 1 mole of a Van der Waals gas. 8

Group B

Answer any two questions

7. a)Distinguish between inertial and non-inertial frames of reference. 6

b)Show that in the northern hemisphere, the Coriolis force deflects a freely falling body of mass m towards the east and has a magnitude $2mv\omega\cos\lambda$, where v is the magnitude of the downward velocity at latitude λ and ω is the angular velocity of the earth. 3

c) State parallel axis theorem. A sphere of radius 2r and density ρ has a spherical cavity of radius r. The diameter of the cavity coincides with a radius of the sphere. Find the moment of inertia of the body about an axis perpendicular to the common diameter of the sphere and the cavity and passing through the centre of the cavity. Assume the moment of inertia of a sphere is known. 4+8

d)State Stoke's law for terminal velocity of a sphere falling through a viscous fluid maintaining an irrotational flow past it. Write down the equation of motion of the falling sphere subjected to a viscous force proportional to the instantaneous velocity of the sphere. Solve the equation to obtain an expression for terminal velocity. 4+10

8. a) Show that four velocity is time-like while four acceleration is space-like. 6

b) A thin equiconvex lens of refractive index 1.6 and radii of curvatures 12 cm is silvered on a side opposite to the direction of incident beam. Obtain the system matrix and find the power of this system for light entering the unsilvered side. 6

c)Compare Fresnel's diffraction at a circular aperture and a circular obstacle.
A circular aperture of 1.4×10^{-3} m diameter is illuminated by plane wave of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The centre of the circular path of light first becomes dark when the screen is 0.49 m from the aperture. Calculate the wavelength of light. 5 + 5

d)State Huygen's theory of double refraction in uniaxial crystals.
The refractive index of calcite for ordinary ray is 1.6584, the refractive index of Canada balsam is 1.550. A ray of light is incident parallel to the length side. Calculate the angle of incidence of the ordinary ray on the Canada balsam layer and show that it will suffer total internal reflection at that layer. 6+6

Contd...P/5.

e)The bandwidth of the He-Ne laser is 500 Hz. Determine the coherence time and the longitudinal coherence length. 6

9. a) Find the electric field inside a sphere which carries a charge density proportional to the distance from the origin $\rho = kr$, for some constant k. 6

b) State Biot-Savart's law and calculate the electric field at an axial point of an infinite solenoid of radius 'a'. 4+10

c)A coil of resistance 60 Ω and inductance 3 H is connected in series with a capacitor 4 μF and an a.c. supply of 200 V and 50 Hz. Calculate the impedance of the circuit and the phase difference between current and voltage. 8

d) Find the Lagrangian and the Lagrange's equation of motion of a pendulum whose point of suspension is attached to a lift that falls with an acceleration f. 12

10. a)A gas occupies 1 litre at 80 cm pressure. It is expanded adiabatically to 1190 cc. If the pressure falls to 60 cm in the process, find the ratio of specific heat. 6

b)Prove that no engine can be more efficient than a reversible one working between the same two temperature limits. 6

c)Deduce the relation $(\frac{\partial Q}{\partial V})_T = T(\frac{\partial P}{\partial T})_V$ and hence derive Clausius-Clapeyron latent heat equation. 6+8

d)Define coefficient of viscosity and discuss how it depends on temperature. What is the S.I. unit of coefficient of viscosity ?
Three capillaries of same length but different internal radii of values 3a, 4a and 5a are connected in series and a liquid flows through them in streamlined condition. If the pressure difference across the third capillary is 8.1 mm, find the pressure difference across the first capillary. 8+6