2017

MATHEMATICS - PAPER-II

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali but all answers must be in one and the same language.

GROUP - A

Answer any five questions

1. (a) Find the roots of $z^n = (1 + z)^n$ and show that points which represent them are Colinear.

   (b) If p be a prime and be not a divisor of a, then prove that $a^{p-1} \equiv 1 \pmod p$. If $ac \equiv bc \pmod m$ and $(c, m) = 1$, then show that $a \equiv b \pmod m$. 10 + 4

   10 + 4

2. (a) If α be a special root of the equation $x^8 - 1 = 0$ then prove that $(\alpha + 2)(\alpha^2 + 2) \ldots (\alpha^7 + 2) = 85$.

   (b) a, b, c are three +ve number in Harmonic Progression. Prove that $a^n + c^n > 2b^n$, $n > 1$, n is a natural number. 14 + 14

3. (a) If in a group G, $a^5 = e$, $aba^{-1} = b^2$ for $a, b \in G$, find $o(b)$.

   (b) Prove that intersection of two subrings is a subring. 14 + 14

4. (a) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, prove that -

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right).$$

   (b) Evaluate $\iint_R (x^2 + y^2)\, dx\, dy$ over R bounded by $y = x^2$, $x = 2$, $y = 1$. 14 + 14

5. (a) If r be the distance of $p(x,y,z)$ from the origin and $\vec{r}$ be the position vector of P relative to the origin, then find $\nabla^2\left(\frac{1}{r}\right)$.

   (b) If $\vec{A} = 3xy\,\hat{i} - 5z\hat{j} + 10x\hat{k}$, then evaluate $\int \vec{A}\cdot d\vec{r}$ along the curve C given by $x = t^2+1$, $y = 2t^2$, $z = t^3$ from t=1 to t=2.

   14 + 14

6. (a) Let x be the set of all real-valued continuous functions f(x) defined on a closed interval $[a,b]$. If $f(x), g(x) \in X$, we define a mapping

$$d : X \times X \to R \text{ by the rule}$$

$$d(f, g) = \sup_{a \le x \le b} |f(x) - g(x)|.$$

   Show that (x, d) is a metric space.

   (b) Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function. 14 + 14

7. (a) Find the root of $x^2 + \ln x - 2 = 0$, which between 1 and 2, by fixed point iteration method, correct to three decimal places.

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(b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y(o)=1, find y approximately, for x=0.1, taking h=0.01, correct to 4-significant figures by Euler's method. 14 + 14

8. (a) Find the disjunctive normal form of $[x'.y+(x.z)'].(x+y.z)'$. Convert 77_{10} into binary number system. 10 + 4

(b) Design an algorithm and draw a flow chart to evaluate the definite integral $\int_a^b f(x)\,dx$ by Simpson's one-third rule. 7 + 7

GROUP - B

Answer any two questions

9. (a) Find the mean, variance and the coefficient of skewness of the continuous distribution with probability density function given by

$$f(x) = 1 - |1-x|, \quad 0 < x < 2$$
$$= 0 \quad , \text{ elsewhere.}$$
6 + 6 + 4

(b) The random variables x and y have the joint density function

$$f(x,y) = \begin{cases} 6(1-x-y), & x>0,\ y>0,\ x+y<1 \\ 0 & , \text{ elsewhere} \end{cases}$$

Find the marginal distribution of x and y. Are x and y independent ? 12 + 2

10. (a) Find the least square parabolic fit of the form $y=ax^2+bx+c$ to the following data -

x	-3	-1	1	3
y	15	5	1	5

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(b) Find the maximum likehood estimate of θ in $f(x)=y_0 e^{-\frac{x}{\theta}}$; $x>0$, $\theta>0$ on the basis of a random sample of size n. 14

11(a) Solve the following L.P.P.

$$\text{Maximize } z = 3x_1 + 4x_2$$
$$\text{Subject to } x_1 + x_2 \le 10$$
$$2x_1 + 3x_2 \le 18$$
$$x_1 \le 8$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

by solving its dual problem. 16

(b) Solve the assignment problem where the assignment cost of assigning any operator to any one machine is given below :-

Operators

Machines	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

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