BM-(U)ST-1/17

2017

STATISTICS – PAPER-I

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be written either in English or in Bengali but all answers must be in one and the same language.

GROUP – A

Answer any three questions

1. (a) For any two events A and B show that –

$$\max\{0,\ P(A)+P(B)-1\} \leq P(A\cap B) \leq \min\{P(A),\ P(B)\} \leq \max\{P(A),\ P(B)\} \leq P(A\cup B) \leq \min\{P(A)+P(B),\ 1\}.$$

(b) Assume that n random variables $x_1, x_2, \ldots, x_n$ are independent and each takes the values +1 and −1 with probabilities p and q=1−p respectively. Find the expectation and variance of the product of the random variables.

(c) State and prove Bayes' theorem. 8 + 12 + 10

2. (a) Let x be any random variable with distribution function F(x) and assume E(x) exists. Show that –

$$E(X) = \int_0^{\infty}\left[1 - F(x) + F(-x)\right] dx.$$

(b) Define a quartile based measure of dispersion and find such a measure for a random variable with pdf

$$f(x) = \begin{cases} x^{\frac{\alpha}{\alpha+1}}, & x \geqslant 1,\ \alpha > 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) You are given that at least one of the events A_r, $r = 1,2,\ldots,n$ is certain to occur, but certainly no more than two occur. If $P(A_r) = p$ and $P(A_r \cap A_s) = q$, $r \neq s$, show that $p \geqslant \frac{1}{n}$ and $q \leqslant \frac{2}{n}$.

[10 + 10 + 10]

3 (a) Let X and Y jointly follow a bivariate normal distribution with means 0, variances 1 and correlation coefficient $\rho\ (-1 < \rho < 1)$. Prove that –

(b) Let $(X_1, X_2, X_3) \sim N_3\left(0, \begin{pmatrix} 1 & P_{12} & P_{13} \\ P_{12} & 1 & P_{23} \\ P_{13} & P_{23} & 1 \end{pmatrix}\right)$.

Show that

$$P(X_1 > 0, X_2 > 0, X_3 > 0) = \frac{1}{8} + \frac{\sin^{-1}P_{12} + \sin^{-1}P_{13} + \sin^{-1}P_{23}}{4\pi}$$

P. T. O.

(c) Prove that for a set of p variables, if all the simple correlation coefficients are negative, then all the partial correlation coefficients of all orders are also negative.

10 + 10 + 10

4. (a) Derive the likelihood ratio test for equality of means of $k (\geq 2)$ independent normal populations. If the test is significant what would be your next steps ?

(b) Describe an exact test of significance for the ratio of two normal variances. Consider both correlated and uncorrelated cases.

15 + 15

5. Write notes on :-

(a) p - value

(b) Randomized and non-randomized tests.

(c) Analysis of Variance.

10 + 10 +10

GROUP - B

Answer any two questions

6. (a) In stratified simple random without replacement sampling find the optimum allocation which minimizes cost given a fixed level of efficiency assuming a linear cost function.

(b) Find approximate expressions of the bias and mean square error of the ratio estimator of the population mean in SRSWOR.

20 + 20

7. Suppose we want to judge whether the grades given by five teachers for the same course for five different sections are same.

(a) Stating your assumptions clearly, set up an analysis of variance model for the problem.

(b) Write down the hypothesis you may like to test.

(c) Derive the test statistic and describe the test procedure.

(d) What further analysis would you carry out if the null hypothesis is rejected ?

10+6+14+10

8. (a) State fundamental Neyman-Pearson lemma and prove its sufficiency part.

(b) Discuss sign test and signed rank test explaining their applications.

(c) What do you mean by combination of probabilities in tests of significance ?

16 + 16 + 8