2018

MATHEMATICS - PAPER-I

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali but all answers must be in one and the same language.

1. Answer any two questions :-

(a) Find a basis and dimension of the subsperce W of R^3, where $W = \{(x,y,z) + R^3 : x+2y+z=0,\ 2x+y+3z=0\}$

(b) The matrix of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the ordered basis $((0,1,1), (1,0,1), (1,1,W))$ of $\mathbb{R}^3$ is given by $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$ Find T.

Find the matrix of T relative to ordered basis $((2,1,1), (1,2,1), (1,1,2))$ of $\mathbb{R}^3$.

(c) If $p(x)$ is a polynomial of degree >1 and $K \in \mathbb{R}$, prove that between two real roots of $p(x) = 0$ there is a real root of $p'(x) + kp(x) = 0$.

10 x 2

2. Answer any two questions :-

(a) A sequence $\{u_n\}$ is defined by $u_n > 0$ and $un+1 = \frac{6}{1+u_n}\ \forall n \in \mathbb{N}$

(i) Prove that the subsequence $\{u_{2n-1}\}$ and $\{u_{2n}\}$ converge to the same limit.

(ii) Find $\lim_{n\to\infty} U_n$

(b) Test the convergence of the series $\sum U_n$ where $U_n = \sqrt{n^4+1} - \sqrt{n^4-1}$

(c) Find $\lim_{n\to\infty} (1 + \frac{1}{n})^{2n}$

10 x 2

3. Answer any two questions :-

(a) Prove that $\frac{\pi^2}{9} < \int_{\pi/6}^{\pi/2} \frac{x}{\sin x}\,dx < \frac{2\pi^2}{9}$

(b) Is $f_n(x) = \frac{nx}{1+n^2x^2}$ uniformly convergent on $[0,1]$? Justify it.

(c) Find the radius of convergence of the power series

$x + \frac{x^2}{2!} \cdot 2^2 + \frac{x^3}{3!} 3^3 + \ldots\infty$

10 x 2

4. Answer any two questions :-

(a) Determine the condition for which the system of equations

$$\left.\begin{array}{l} x + y + z = 1 \\ x + 2y - z = b \\ 5x + 7y + az = b^2 \end{array}\right\}$$

admits of (i) only one Solution (ii) no solution (iii) many solution.

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(b) Find the asymptotes of the given curve $x^3+y^3-3xy = 0$

(c) Find the pedal equation of the following curve $r^2=a^2\sin 2\theta$. 10x2

5. Answer any two questions :-

(a) Show that the triange formed by st.lines $ax^2 + 2hxy + by^2 = 0$ and the st.line $lx+my=1$ is right angled if $(a+b)(al^2+2hlm+bm^2)=0$

(b) If a point lies on the ellipse $\frac{x^2}{a'^2} + \frac{y^2}{b'^2} = 1$, prove that its polar w.r.t. the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the ellipse $\frac{a'^2}{a^4}x^2 + \frac{b'^2}{b^4}y^2 = 1$

(c) Show that the product of focal distances of a point on an ellipse is equal to the square of the length of semi-diameter parallel to the tangent at this point. 10x2

6. Answer any two questions :-

(a) The plane $lx+my=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α. Prove that in its new position, its equation will be $lx+my \pm z\sqrt{l^2+m^2}\tan\alpha = 0$.

(b) If the lines $x = ay + b = cz + d$, $x = \alpha y + \beta = \gamma z + \delta$ are co-planar, then show that $(\gamma - c)(\alpha\beta - b\alpha) = (a-\alpha)(c\delta - d\gamma)$.

(c) Show that the locus of a point which is equidistant from the given straight lines $y = mx$, $z=c$ and $y=mx$, $z=-c$ is $mxy + c(1+m^2)z = 0$. 10x2

7. Answer any two questions :-

(a) Solve : $y^2\log y = xpy + p^2$

(b) Solve : $(D^2-3D+2)y = e^{3x}$

(c) Solve : $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$ 10x2

8. Answer any two questions :-

(a) Solve $x^2p + y^2q = (x+y)z$

(b) Solve by Charpit method $(x^2-y^2)pq - xy(p^2 - q^2) = 1$

(c) Find the complete integral of $z^2 = pqxy$

(p, q have their usual significance) 10x2

9. Answer any two questions :-

(a) The moments of a system of forces about the points (0,0), (a,0), (0,a) are aw, 2aw, 3aw respectively. Find the components of their resultants parallel to the co-ordinate axes and the equation to its line of action.

(b) Two equal ladders of weight w are placed so as to lean against each other at an angle 2θ, with their ends resting on a rough horizontal floor, the coefficients of friction of which w.r.t. either being μ, then show that $\tan\theta > \mu > \frac{1}{2}\tan\theta$.

(c) If a force (X, Y, Z) act along a generator of the hyperbolic parabolid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$ and be equivalent to an equal forces (X,Y,Z) at the origin together with a couple (L,M,N), show that $aL \pm bM = 0$, $bX \pm aY = 0$ and $cN \pm abZ = 0$ 10x2

10. Answer any two questions :-

(a) A particle moving in a straight line is acted on by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is

$$\frac{3(u+v)x}{2(u^2+uv+v^2)}$$

(b) Two perfectly inelastic bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy due to impact is $\frac{1}{2} \frac{m_1 m_2}{m_1+m_2} (u_1 - u_2)^2$

(c) A particle is executing a S.H.M. in a straight line under an accleration μx(distance). If a periodic force mL Cos pt be introduced and the time period of force vibration increased to $1\frac{1}{2}$ times, show that $9p^2 = 4\mu$.

10x2

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