CS(O) MA-II /18

2018

MATHEMATICS - PAPER-II

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali but all answers must be in one and the same language.

GROUP - A

Answer any five questions

1. (a) The roats of the equation $z^2 + pz + q = 0$, where p, q are complex numbers, are represented by the points A, B on the complex plane. If OA=OB and $\angle AOB = 2\beta$, where O is the origin, prove that -

$$p^2 = 4q\cos^2\beta$$ 14

(b) Prove that $3.4^{n+1} \equiv 3 \pmod 9$ for all positive integer n. Find the remainder when 1 ! + 2 ! + 3 ! + ... + 50 ! is divided by 15. If p be a prime number greater than 2, prove that

$$1^p + 2^p + \ldots + (p-1)^p \equiv 0 \pmod p.$$ 4 + 5 + 5

2. (a) If the product of two roots of the equation $x^4+px^3+qx^2+rx+s = 0$ be equal to the product of the other two, prove that $r^2=p^2s$. If $p \neq 0$, show that the equation can be solved by the substitution $x + \frac{r}{px} = t$. 10 + 4

(b) If n be a positive integer, prove that

$$\frac{1}{\sqrt{4n+1}} < \left\{\frac{3.7.11.\ \ldots\ (4n-1)}{5.9.13.\ldots.\ (4n+1)}\right\} < \sqrt{\frac{3}{4n+3}}$$ 14

3. (a) Show that if G is a group of order 10, then it must have a subgroup of order 5. Suppose G is a finite group of order pq, where p, q are primes and p> q. Show that G has at most one subgroup of order p. 8 + 6

(b) If G is a group such that $\frac{G}{Z(G)}$ is cyclic, where Z(G) is the center of G, then show that G is abelian. 14

4. (a) Given that $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2 \neq 0 \\ 0, & \text{when } x^2+y^2=0 \end{cases}$

Prove that f is continuous at (0,0) and f_x, f_y both exist at (0,0) but f is not differentiable at (0,0). 4 + 5 + 5

(b) Prove that the functions u, v, w given by $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$ and $w = \frac{z}{x-y}$ are linearly independent. But u, v , w are related by the relation $uv + vw + wu = -1$

5. (a) If $\vec{a}$ is a constant vector, then prove that

$$\text{curl}\ \frac{\vec{a} \times \vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a}.\ \vec{r}) \quad \text{where } r = |\vec{r}|$$ 14

(b) Evaluate $\iint_S \vec{F}.\ \vec{n}\ ds$, where $\vec{F} = y\hat{i} + zx\hat{j} - z\hat{k}$ and S is the surface of the plane $2x + y = 6$ in the first octant cut off by the plane $z = 4$. 14

P. T. O.

6. (a) Let x be the set of all real valued continuously differentiable functions on a, b and d is defined on Xx X by

$$d(f,g) = \sup_{a \leq x \leq b} \left| f(x) - g(x) \right| + \sup_{a \leq x \leq b} \left| f'(x) - g'(x) \right|,$$

for f, g ∈ X ; prove that (x, d) is a metric space. 14

(b) Show that $u(x,y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate. 6 + 8

7. (a) If $y = x^3+x^2-2x+1$, calculate the values of y for x = 0,1,2,3,4,5 and form the difference table. Find the value of y at x=6 by extending the table and verify that the same value is obtained by substitution. 14

(b) Obtain a real root of the equation $e^x - 3x = 0$, by Newton-Rapson method, correct to four decimal places. 14

GROUP - B

Answer any two questions

8. (a) Let X and Y be two continuous random variables having joint probability density function

$$f(x,y) = 1 - \frac{x}{3} - \frac{y}{3}, \quad 0 \leq x \leq 2, \ 0 \leq y \leq 1$$
$$= 0, \text{ otherwise.}$$

Obtain the marginal densities of X and y. Also find

$E(xy)$, $P(x < y)$ and $P(x \geq 1, y \leq 0.5)$ 5+5+5+5

(b) If the probability density function of a random variable x is given by

$$f(x) = c\, e^{-(x^2+2x+3)} \quad -\infty < x < \infty$$, find the value of C

and the variance of the distribution. 5 + 5

9. (a) Show that the correlation coefficient between two random variables lies between 1 and -1. 15

(b) Out of two regression lines given by x + 2y = 5 and 2x + 3y = 8 which one is the regression line of x on y ?

Find also the values of $\bar{x}$, $\bar{y}$, r and σ_y, given that $\sigma_x = 12$.

5+2.5+2.5+2.5+2.5

10. (a) Apply the two-phase process to show that the following L.P.P has unbounded solution :

Max. $Z = 2x_1 - x_2 + x_3$

Subject to $x_1 + x_2 - 3x_3 \leq 8$

$4x_1 - x_2 + x_3 \geq 2$

$2x_1 + 3x_2 - x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$ 15

(b) Find the optimum assignment to find the minimum cost for the assignment problem with the following cost matrix :-

	M_1	M_2	M_3	M_4	M_5
J_1	8	4	2	6	1
J_2	0	9	5	5	4
J_3	3	8	9	2	6
J_4	4	3	1	0	3
J_5	9	5	8	9	5

15