2018

# STATISTICS-PAPER-I

Time Allowed : 3 Hours

Full Marks : 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali or in Nepali but all answers must be in one and the same language.

## GROUP - A

Answer any ten questions.

1. (a) What do you mean by 'statistical regularity' ? Define probability space and state Kolmogorov's axiomatic definition of probability.

(b) What do you mean by a symmetric probability distribution ? If X be a continuous random variable having distribution function F satisfying

$$F(x) + F(-x) = 1 \quad \text{for all } x$$

then show that $E(X)=0$ assuming $E(x)$ exists.

(c) Write down the properties of a distribution function. If F is a distribution function then show that for any natural number n $F^n$ will also be a distribution function.

(d) Define best linear unbiased estimator (BLUE). Show that sample mean is the BLUE of population mean.

(e) Define a likelihood function and a maximum likelihood estimator (MLE) of a parametric function. Suppose $(x_1, x_2, \ldots, x_n)$ is a random sample drawn from uniform$(0, \phi)$. Find MLE of $\phi$.

(f) Distinguish between critical value and p-value of a test.

(g) What do you mean by a most powerful test ? In this connection state fundament Neyman-Pearson Lemma.

(h) Define convergence in probability and convergence in distribution. Cite an example to show that the latter does not imply the former.

(i) Define multiple correlation coefficient and show that it always lies in the interval $[0, 1]$. Also interpret the cases when it assumes 0 and 1.

(j) Suppose $\underset{\sim}{X}$ is a p-component random vector with mean vector $\underset{\sim}{\mu}$ and nonsingular dispersion matrix $\Sigma$. Then show that

$$P\left( (\underset{\sim}{x} - \underset{\sim}{\mu})' \Sigma^{-1} (\underset{\sim}{x} - \underset{\sim}{\mu}) > \lambda \right) < \frac{p}{\lambda}, \text{ where } \lambda > 0$$

(k) What are the basic principles of survey sampling ? Discuss.

(l) Distinguish between linear and circular systematic sampling.

(m) What are the minimal assumptions in analysis of variance technique ? Describe Gauss Markov model in this connection.

(n) What are uniformity trial data in an experimental design ? Write down the role of such a data set.

(o) Obtain the relative efficiency of a latin square design as compared to a completely randomised block design. 10x10

## GROUP - B

Answer any five questions.

2. (a) Using axiomatic definition of probability show that $P(0)=0$ and $P(A \cup B) = P(A) + P(B)$, where $A \quad B =$

(b) Distinguish between pairwise independent and mutually independent events. Cite an example to show that the former does not imply the latter.

(c) Two fair dice are rolled together $r\ (>6)$ times. Find the probability that each of the cases $(i, i)$, $i = 1,2,\ldots,6$ will occur atleast once.

(3+3)+(4+3)+7

3. (a) State and prove Chebyshev's inequality.

(b) Suppose $\{X_n\}$ is a sequence of independent random variables having probability distribution

$$P(X_n = -\frac{1}{2^n}) = \frac{1}{2} = P(X_n = \frac{1}{2^n}),\ n=1,2,3,\ldots.$$

Show that the sequence obeys weak law of large numbers.

(c) What is central limit theorem ? Write down its applications citing examples.

6+6+(2+6)

4. (a) Suppose $x_1, x_2, \ldots\ldots x_n$ is a random sample drawn from a continuous population having pdf

$$f_{\alpha,\theta}(x) = \begin{cases} \frac{1}{\alpha} e^{-\left(\frac{x-\theta}{\alpha}\right)} & \text{if } x > \theta \\ 0 & \text{elsewhere} \end{cases}$$

Obtain sufficient statistic of $(\alpha, \theta)$.

(b) What do you mean by Blackwellisation in a problem of point extimation ?

(c) Suppose $X_1, X_2, \ldots, X_n$ is a random sample drawn from a Poisson( $\lambda$ ) population. Then find maximum likelihood estimator of $e^{-\lambda}$.

7+5+8

5. (a) Define uniformly most accurate confidence sets. How is it related to a uniformly most powerful test ?

(b) Obtain most powerful size $\alpha$ test based on a single observation $X$ to test.

$H_0$ : X follows standard normal

Versus

$H_1$ : X follows standard Cauchy.

(4+6)+10

6. (a) If $\underset{\sim}{X}' = (x_1, x_2, \ldots x_p)$ has a multivariate normal distribution with mean vector $\underset{\sim}{0}$ and dispersion matrix $I_p$. Then find the distribution of $\underset{\sim}{X}'\underset{\sim}{X}$ and $(x_1, x_2, \ldots\ldots x_m), m < p$.

(b) If $\underset{\sim}{X}' = (x_1, x_2, \ldots, x_p)$ has a positive definite dispersion matrix, then for a matrix $B^{r \times p}$ ( $r < p$ ) with full row rank, find the dispersion matrix of $B\underset{\sim}{X}$. Comment whether the resulting distribution will be singular or not.

(c) State and prove sum law of expectation with respect to a multivariate probability distribution.

(4+4)+(6+2)+4

7. (a) A simple random sample of size $n$ is drawn without replacement from a finite identifiable population with variate values $Y_1, Y_2, \ldots\ldots, Y_N$. Find unbiased estimator of $\sum_{\alpha=1}^{N} Y_\alpha$ and $\sum_{\alpha \in I} Y_\alpha$ together with the unbiased estimator of the variance of the estimators, where I consists of population units possessing a character 'A'.

(b) What do you mean by intra systematic sample correlation coefficient ($\rho_s$)? Obtain an unbiased estimator of population mean based on a linear systematic sample. Find the variance of the estimator in terms of $\rho_s$ and comment.

12+(2+2+3+1)

8. Describe in detail analysis of variance technique in two way classified data with equal number of observations per cell. In this context define valied error. Here, if observed value of F statistic is less than unity we accept the null hypothesis of no difference among the effects trivially - Justify.

12+4+4

9. (a) Describe the analysis of a $2^3$ experiment conducted in an RBD with randomised blocks.

(b) What do you mean by an observational contract and a treatment contract ? Show that in an RBD a block contract is orthogonal to a treatment contract.

10+(4+6)

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