2019

Time Allowed- 3 Hours

STATISTICS– PAPER-I

MSC(O)ST -I/19

Full Marks – 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group – A

Answer any ten questions

1. (a) Give the classical definition of probability. What are its limitations? Justify it with examples. 3+3+4

(b) (i) An urn contains one black ball and one green ball. Another urn contains one white ball and one green ball. One ball is selected from each urn. What is the probability that both the balls will be of the same colour? Also get the probability of getting one green ball.

(ii) Find the probability of getting at least 2 heads if a fair coin is tossed six times independently. 5+1+4

(c) For any two events A and B, prove that

$$P(A) + P(B) - 1 \leq P(A \cap B) \leq \sqrt{P(A)\,P(B)}.$$

Discuss the case for equality. 8+2

(d) Distinguish between total independence and pairwise independence for a set of events. Illustrate with an example. 6+4

(e) Define random variable and its distribution function. Discuss the important properties of a distribution function. 3+3+4

(f) Define expectation of a discrete random variable. For any two such variables, X and Y, prove that

$$E(x+y) = E(x) + E(y).$$

When does the result

$$E(xy) = E(x)\,E(y)$$

hold? 7+3

(g) Show that $E|x-c|$ is minimised at c=Median(x). 10

(h) When is discrete distribution said to be symmetric? For such a distribution, discuss the relation between mean, median and mode. 4+6

(i) State and prove Chebyshev's inequality. Discuss one of its uses. 6+4

(j) Discuss the properties of a bivariate normal distribution. 10

(k) (i) What is a sufficient statistic in point estimation? State a necessary and sufficient condition related to this statistic.

(ii) Let X be $N(\mu, 1)$ with unknown μ. Show that $|x|$ is not sufficient for μ. 4+4+2

(l) Discuss the role of auxiliary information in survey sampling. Mention at least three methods where such information is used. 5+5

P.T.O.

(m) What are random sampling numbers? Mention some important random sampling number series and discribe their methods of construction. 4+6

(n) Mention the basic principles of design of experiments. Provide two designs in which all the basic principles are used. 6+4

(o) Define the terms main effects and interaction effects in relation to a 2^3 experiment. Show that they are all treatment contrasts orthogonal to each other. 6+4

Group - B

Answer any five questions

2. (a) For any two events, A and B, define conditional probabilities and prove that (with appropriate assumption)

$$P(A|B^c) = \frac{P(A)[1-P(B/A)]}{P(B^c)}$$

(b) Show that independence of A and B is equivalent to that of A^c and B. 12+8

3. (a) Consider the distribution of (x_1, x_2, x_3) which assumes the values (1,0,0), (0,1,0), (0,0,1) and (1,1,1) each with probability $\frac{1}{4}$. Examine the independence of three random variables.

(b) Let X be distributed according to a p-variate normal distribution with mean vector μ and dispersion matrix Σ, which is positive definite. Prove that the components of X are independent if and only if Σ is a diagonal matrix. 10+10

4. (a) State the central limit theorem for independently and indentically distributed random variables and use it to find the asymptotic districution of (i) X following binomial (n,p), $0<p<1$, and (ii) X following chi-square distribution with n degrees of freedom.

(b) Let X_n be a sequence of random variables with

$$\underline{P}(x_n = 1) = \frac{1}{n}, \quad \underline{P}(x_n = 0) = 1 - \frac{1}{n}, \quad n \geq 1.$$

Examine whether X_n converges to zero in probability. 15+5

5. (a) Discuss the properties of a good estimator.

(b) Let $\{x_1, \ldots x_n\}$ be a random sample with mean μ and variance σ^2. Find C such that

$$C \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2$$

is an unbiased estimator of σ^2 [Here random sample means that $x_1 \ldots x_n$ are iid].

(c) Explain, with an illustration, the concept of pivot in confidence interval estimation. 5+8+7

6. (a) With reference to problem of testing hypothesis, explain the concepts of type I error, type II error, level of significance and power of a test.

(b) Let x be uniform $[\theta, \theta+1]$, $\theta \geq 0$. Find a most powerful test of size α $(0 < \alpha < \frac{1}{2})$ based on single observation (X) for $H_0 : \theta = 0$ against $H_1 : \theta = \frac{1}{2}$.

Derive the power function of this test.

12+8

7. Let $(x_1, \ldots\ldots x_n)$ be a sample from $N(\mu, \sigma^2)$ with unknown μ and σ^2.

(a) Find the maximum likelihood and moment estimators of (μ, σ^2). Are they identical? Comment.

(b) Show that the estimators are consistent.

12+8

8. (a) Distinguish between two-stage sampling and stratified random sampling.

(b) For a two-state sampling, where the first stage units are of equal size, obtain an estimator of the population total. Also obtain the expression for the variance of the estimator. How will you estimate the variance from your sample?

5+15

9. (a) Starting from an appropriate linear model, prepare the ANOVA table for analysis of 6x6 Latin Square Design.

(b) Indicate the analysis of a 2^4 in a single replicate.

(c) What is analysis of covariance? Write down the appropriate linear model with estimators of treatment effects in Randomised Block Design.

6+6+8
