2019

MATHEMATICS

PAPER-II

Time Allowed — 3 Hours

Full Marks — 200

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in **English** *or in* **Bengali** *but all answers must be in one and the same language.*

Group A

Answer any five *questions.*

1. (a) If the equation $x^4 + px^3 + qx^2 + rx + s = 0$ has roots of the form $\alpha + i\alpha$, $\beta + i\beta$, where α, β are real. Prove that $p^2 - 2q = 0$ and $r^2 - 2qs = 0$. 14

   (b) Prove that $\sqrt{n} < \sqrt[n]{n!} < \frac{n+1}{2}\ \forall n > 2$. 14

2. (a) If p is an odd prime prove that

   (i) $1^2.3^2.5^2 \dots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} (mod\ p)$

   (ii) $2^2.4^2.6^2 \dots (p-1)^2 \equiv (-1)^{\frac{p+1}{2}} (mod\ p)$ 7+7=14

   (b) (i) Show that the roots of the equation $(1+z)^n = (1-z)^n$ are the values of $i\tan\left(\frac{r\pi}{n}\right)$, where $r = 0, 1, 2, \dots n-1$, but omitting $\frac{n}{2}$ if n is even.

   (ii) Show that the product of all values of $(\sqrt{3} + i)^{\frac{3}{5}}$ is $8i$. 7+7=14

3. (a) Define f over R^2 by

$$f(x,y) = \begin{cases} \left(\frac{|x|}{y^2}\right).e^{\frac{-|x|}{y^2}} & ,y \neq 0 \\ 0 & ,y = 0 \end{cases}$$

   Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ exists along any straight line but the limit does not exist. 14

   (b) Let $f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$

   Show that both f_x and f_y exist at (0, 0) but f is not differentiable at (0, 0). 7+7=14

4. (a) If V is a closed region bounded by the planes $x = 0, y = 0, z = 0, 2x + 2y + z = 4$ and $F = (3x^2 - 8z)i - 2xyj - 8xk$, then show that

   (i) $\iiint_V \nabla.F dV = \frac{16}{3}$

   (ii) $\iiint_V \nabla \times F d\nabla = -\frac{8}{3}k$ 7+7=14

(b) Proved that $a.\nabla\left(b.\nabla\left(\frac{1}{r}\right)\right) = \frac{(3(a.\vec{r})(b.\vec{r}))}{r^5} - \frac{a.b}{r^3}$ where a and b are constant vectors and $r = |\vec{r}|$. 14

5. (a) Let $(G, \circ)$ be a group and $(H, \circ)$ be a subgroup of $(G, \circ)$. Let $x, y \in G$ and a relation ρ is defined on G by "$x \rho y$ iff $x \circ y^{-1} \in H$." Prove that ρ is an equivalence relation on G. 14

(b) If an abelian group G of order 10 contains an element of order 5, prove that G must be cyclic group. 14

6. (a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$. Find $f(z)$ in terms of z. 14

(b) Suppose X is a non-empty set and $d(a, a) = 0$ for all $a \in X$ *and* $d(a, b) = 1$ for all $a, b \in X$ with $a \neq b$. Show that d is a metric on X. 14

7. (a) Construct the Lagrange interpolation polynomial for the data.

x	-1	1	4	7
$f(x)$	-2	0	63	32

Hence interpolate at $x = 5$. 14

(b) Using Newton–Raphson method solve $x \log_{10} x = 12 \cdot 34$ with $x_0 = 10$. 14

Group B

Answer any two *questions.*

8. (a) If the independent random variables X and Y be each uniformly distributed in the interval $(-a, a)$ then find the distribution of

(a) $X + Y$

(b) XY

(c) $\frac{X}{Y}$ 5+5+5=15

(b) In the equation $x^2 + 2x - q = 0, q$ is a random variable uniformly distributed over the interval (0, 2). Find the distribution function of the largest root. 15

9. (a) The least square regression lines of Y on X and X on Y are respectively $x + 3y = 0, 3x + 2y = 0$. If $\sigma_x = 1$ then find the least square regression line of V on U where $U = X + Y, V = X - Y$. 15

(b) Let $U = aX + bY$ and $V = bX - aY$. If $E(X) = E(Y) = 0$ and if $\rho(X, Y) = \rho, \rho(U, V) = 0$, then show that

(i) $\text{var}U.\text{var}V = (a^2 + b^2)^2\,(\text{var}X)(\text{var}Y)(1 - \rho^2)$

(ii) $ab(\text{var}X - \text{var}Y) = \rho\sigma_x\sigma_y(a^2 - b^2)$ 7·5+7·5=15

10. (a) By solving the dual of the primal problem:

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 7,$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$4x_1 + 7x_2 - 2x_3 \geq 2$

$x_1 - 2x_2 + 5x_3 \geq 3, \quad x_1, x_2, x_3 \geq 0,$

Show that it has no solution.

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(b) Five operators (A, B, C, D, E) have been assigned to five machines (I, II, III, IV, V). Operator A cannot operate machine III and operator C cannot operate machine IV. Find the optimal assignment schedule.

	I	II	III	IV	V
A	5	5	-	2	6
B	7	4	2	3	4
C	9	3	5	-	3
D	7	2	6	7	2
E	6	5	7	9	1

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