2021

MATHEMATICS

PAPER-II

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer any four *questions.*

1. (a) Show that each root of $x^7 - 1 = 0$ is a non negative integral power of $\alpha = \cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}$, $k \in Z$ and also prove that the roots of $x^2 + x + 2 = 0$ are $(\alpha + \alpha^2 + \alpha^4)$ and $(\alpha^3 + \alpha^5 + \alpha^6)$. 5+10=15

   (b) Find the special roots of the equation $x^9 - 1 = 0$ and show that they are roots of the equation $x^6 + x^3 + 1 = 0$. 5+5=10

2. (a) Suppose H is the only subgroup of finite order m in the group G. Prove that H is a normal subgroup of G. If G is a finite group and H is a normal subgroup of G, then prove that $O\left(\frac{G}{H}\right) = \frac{O(G)}{O(H)}$. 10+5=15

   (b) Let Z denote the centre of a group G. If G/Z is cyclic group prove that G is abelian. 10

3. (a) Determine the point where the function is minimum $x^2 + y^2 + (x + y + 1)^2$. 10

   (b) (i) If the vectors $\vec{A}$ and $\vec{B}$ be irrotational, then show that the vector $\vec{A} \times \vec{B}$ is solinoidal.

   (ii) Use Stokes' theorem to evaluate $\oint_C (\cos x dx + 2y^2 dy + z dz)$, where C is the curve $x^2 + y^2 = 1, z = 1$. 5+10=15

4. (a) Let l_p be the set if all real sequences for which $\sum_{i=1}^{\infty} |x_i|^p < \infty$. We define the metric d in l_p by $d(x,y) = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{\frac{1}{p}}$, $\forall x = \{x_i\}$ and $y = \{y_i\} \in l_p$, then prove that the space (l_p, d) is a complete metric space. 15

Please Turn Over

(b) Consider the function f defined by

$$f(z) = 0, \text{when } z = 0$$

$$= \frac{x^3 - y^3}{x^2 + y^2} + i\frac{\left(x^3 + y^3\right)}{x^2 + y^2}, \text{when } z \neq 0.$$

Then show that the function f satisfies the Cauchy-Riemann equation at the origin. 10

5. (a) Prove that $\iint\left\{2a^2 - 2a(x+y) - (x^2+y^2)\right\}dx\,dy = 8\pi dt$. The region of integration being the circle $x^2 + y^2 + 2a(x + y) = 2a^2$. 15

(b) Show that $1!\ 3!\ 5!\ldots\ \ldots\ \ldots(2n-1)! > (n!)^n$. 10

Group-B

Answer any four *questions.*

6. (a) Use Picard's method to compute $y(0{\cdot}1)$, from the differential equation $\frac{dy}{dx} = x + y;\ y = 1$, when $x = 0$, correct to five decimal places. 15

(b) Express the Boolean function $f = x + (x' \cdot y' + x' \cdot z')$ in full disjunctive normal form. 10

7. (a) Find the mean and standard deviation of the following probability distribution $f(x) = Ce^{-\frac{1}{24}(x^2-6x+5)}, -\infty < x < \infty$. Find C. 2½+2½+10=15

(b) If independent random variables X and Y be each uniformly distributed in the interval $(-a, a)$, then find the distribution of

(i) X + Y

(ii) $\frac{X}{Y}$ 5+5=10

8. (a) The first two moments of a sample of size 'n' about the value 4 are 12 and 220 respectively. Find the mean and variance of the observation. 5+10=15

(b) If the relation between variables X and Y, U and V are 2X + 3Y = 4, 3U + 4V = 5 and the regression coefficient of X on U is 4, find the regression coefficient of Y on V. 10

9. (a) Write down the dual of the following L.P.P.

Maximize $Z = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 10$

$2x_1 + 3x_2 \leq 18$

$x_1 \leq 8$

$x_2 \leq 6$

$x_1, x_2 \geq 0.$

Also solve it by using dual problem. 5+10=15

(b) Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 10

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

10. (a) Draw a circuit which realises the Boolean function $f = (x + y).(y + z).(z + x)$. Use the law of Boolean algebra to show that the above circuit is equivalent to a switching circuit in which if any two switches are on, the light is on and construct the equivalent switching circuit. 5+10=15

(b) There are 40 students in Statistics (Hons) class in a particular college. If the habit of borrowing books from college library of a student is $\frac{2}{5}$, find the minimum number of copies of a book, referred in a class, to be kept in the library so to meet more than 90% demand of the students. 10