2021

PHYSICS

PAPER-I

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer *any three* questions.

1. (a) If T be the instantaneous kinetic energy of a moving body of mass m, show that $\frac{d}{dt}(mT) = \vec{F}.\vec{p}$, where $\vec{F}$ is the force acting on the body and $\vec{P}$ is the linear momentum.

   (b) What do you mean by non-inertial frame? Show that the eastward deflection due to effect of coriolis force on a freely falling body at a height 'h' from the earth's surface, initially at rest, is proportional to $h^{3/2}$.

   (c) Show that for a particle moving under a central force, total energy is conserved.

   (d) State the conservation of angular momentum of a system of particles. Show that centre of mass of a system of particles is unique.

   (e) Two particles of masses m_1 and m_2, initially at rest at infinite distance from each other, move under the action of mutual gravitation pull. Show that at any instant their relative velocity of approach is $\sqrt{\frac{2G(m_1+m_2)}{R}}$, where R is the separation at that instant.

   4+(2+10)+8+(3+5)+8=40

2. (a) Mention the advantages of using generalized coordinates in classical mechanics.
   In an inverted pendulum, particle of mass m is attached to a rigid massless rod of length l. If the vertical motion (along Z-axis) of the point of suspension is represented by $Z=a\sin\omega t$, 'a' being a constant, set up the Lagrangian and obtain the equation of motion.

   (b) Write down Euler's equations of motion for a torque-free motion of a rigid body and obtain principle of conservation of total rotational kinetic energy.

   (c) Show the graphical variation of gravitational potential and field inside and outside of a solid sphere (V and E in a single plot).

(d) A half-disc of mass M and radius R rotates about an axis that passes through the centre of the straight side and is perpendicular to its plane. Find moment of inertia about the axis of rotation.

(e) Show that for a cantilever of negligible weight, the depression at any point P due to a load applied at the free end point Q is the same as the depression at Q produced by a similar load at P.

(3+7)+(3+7)+6+6+8=40

3. (a) Write down the postulates of special theory of relativity.

(b) Write down the transformation equations of $\vec{P}$ and E in relativistic mechanics. Assume that the S' frame is moving with a velocity v along x-axis with respect to another inertial frame S.

Prove that $E^2 = p^2C^2 + m_0^2C^4$ is invariant under Lorentz transformation, where the terms have their usual meanings.

(c) A spaceship moving away from the earth with velocity 0·5C fires a rocket whose velocity relative to the space is 0·5C (i) away from the earth, (ii) towards the earth. Calculate the velocity of the rocket as observed from the earth in two cases.

(d) Define group velocity and phase velocity of a wave. A wave packet in a certain medium is constructed by superposing waves of frequency ω arount $\omega_0 = 100$ and the corresponding wave number K with $K_0 = 10$ as given in table below :

ω (s^{-1})	$K(m^{-1})$
81.00	9.0
90.25	9.5
100.00	10.0
110.25	10.5
121.00	11.0

Find the relation between v_g / v_p where v_g is the group velocity and v_p is the phase velocity.

(e) A particle of mass m is oscillating along x-axis under the action of a restoring force proportional to the displacement from the position of equilibrium and a damping force proportional to the instantaneous velocity. Write down the equation of motion of the particle and solve it for the case of critical damping.

4+(6+6)+6+6+(3+9)=40

4. (a) State Fermat's principle in geometrical optics. Obtain laws of refraction at a spherical surface from it.

(b) Obtain focal length of a combination of two thin lenses (foci f_1 and f_2) separated by a distance t using system matrix.

(c) In Newton's ring arrangement, a source emitting two wavelengths $\lambda_1 = 6\times10^{-7}$ m and $\lambda_2 = 5{\cdot}3\times10^{-7}$m. It is found that mth dark ring due to one wavelength coincides with $(m+1)$th dark ring due to the other. If the radius of curvature of the lens is 0·9 m, find the diameter of the mth dark ring.

(d) The central circle of a zone plate has a radius of 0·07 cm. Light of 5000Å coming from an object 147 cm away from the plate falls on it. Find the position of the principal image.

(e) Two linearly polarized light waves are in phase but having different amplitudes, represented by

$$\overrightarrow{E_1}(z,t) = \hat{i}A_1\cos(kz-\omega t) + \hat{j}B_1\cos(kz-\omega t)$$

$$\overrightarrow{E_2}(z,t) = \hat{i}A_2\cos(kz-\omega t) + \hat{j}B_2\cos(kz-\omega t)$$

Find the nature and direction of polarization of $\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2}$. (3+9)+8+6+6+(3+5)=40

5. (a) Obtain Coulomb's law from Gauss's law in electrostatics.

(b) A cylindrical electron beam has a circular cross section of radius a and charge density $\rho(r) = \rho_0\left(1+r^2/a^2\right)$. Find electric field at any internal point ($r<a$).

(c) State Ampere's circuital law. Verify this law for a long straight wire current carrying conductor.

(d) Write down Faraday's law of electromagnetic induction in integral form and obtain its differential form.

(e) An electromagnetic wave in free space with no electric charges or current is represented by $H_x = H_y = 0$ and $H_z = \frac{B}{\mu_0} = A\sin(\alpha y)\cos(\omega t); \alpha =$ a constant. Determine the accompanying electric field $\vec{E}$ and the pointing vectors $\vec{S}$. 6+8+(3+5)+(2+4)+(8+4)=40

6. (a) In a LCR circuit supplied with an ac source of constant angular frequency ω, the value of c is varied continuously. When the current is maximum the value of c is c_0 . When the current falls to $\frac{1}{\sqrt{2}}$ of its maximum value, the values of c are c_1 and c_2. Show that $\frac{1}{c_1}+\frac{1}{c_2}=\frac{2}{c_0}$.

(b) What do you mean by 'entropy' of a system? Prove the relation $Tds = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$, where the terms have their usual meanings.

(c) Mention two difference between reversible adiabatic expansion and Joule-Thomson expansion.

(d) Calculate the variation of C_p with pressure at constant temperature of a substance for which the equation of state is given by $V=\frac{RT}{P}-\frac{C}{T^3}$.

(e) One gm of water vapour at 100°C and atmospheric pressure occupies a volume 1640 cc. Find the vapour pressure of water at 99°C in terms of mm-Hg. Given L=536 cal.

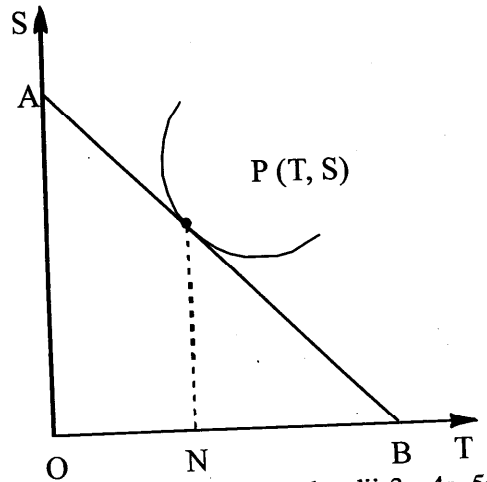
10+(3+5)+4+10+8=40

Group-B

Answer *any two* questions.

7. (a) Define Poisson's ratio. What are the limiting values of it? Why it can not have a negative value?

(b) The plot of surface tension (S) with temperature (T) is shown below with origin O at (T = 0, S = 0). Prove that the intercept of the tangent to the curve with the S-axis at any point is equal to the surface energy per unit area at the corresponding temperature.



(c) Three capillaries of same length but internal radii 3r, 4r, 5r are connected in series and a liquid flows through them in stream line condition. If the pressure difference across the third capillary is 8·1mm, find the pressure difference across the first capillary.

(d) A point mass m is placed on a frictionless plane that is tangent to Earth's surface. Determine Hamilton's equations.

(e) In the Lorentz transformation of space time coordinates, there is a plane in S on which the clock agrees with those of S' after time t. S' is moving with a velocity v with respect to S along the common x-axis. Find the velocity of the plane in S frame.

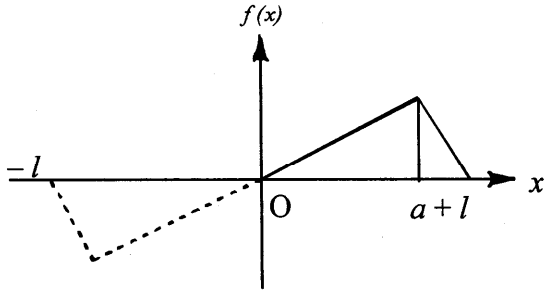
(3+2+2)+10+6+10+7=40

8. (a) Show that two rectangular SHMs of equal frequency but having a phase difference of $\pi/2$ can generate a circular motion.

(b) The vibration of a plucked string is given by

$$f(x)=\frac{hx}{a} \text{ for } 0<x<a$$

$$=\frac{h(l-x)}{(l-a)} \text{ for } a<x<l$$



where a, h, l are all constants as shown in fig.

Find the Fourier analysis of the function.

(c) In Young's double-slit experiment, show that the fringe width for both bright and dark fringes are equal.

(d) Light falls normally on a transparent diffraction grating of width l = 6·5cm with 200 lines per millimetre. The spectrum under investigation includes a spectral line with λ = 670·8nm consisting of two components differing by dl = 0·015nm. Find in what order of the spectrum these components will be resolved.

(e) Obtain the relation connecting Y, η and σ, where the terms have their usual meaning.

6+10+10+6+8=40

9. (a) Obtain mutual potential energy between two dipoles when they lie along the same line on a plane.

(b) Consider a sphere of radius a made of inhomogeneous dielectric whose dielectric constant is given by

$$K = \frac{1}{1+Ar},$$

where A is a constant and r is the radial distance. A charge q is placed at the centre of sphere. Find $\vec{D}$, $\vec{E}$ and $\vec{P}$ at any internal point. Also find the polarization charge densities.

(c) Two similar point charges q, q are kept separated by a distance $2d$ in air. An insulated uncharged conducting sphere of radius a is placed midway between them. If $d>>a$, show that the introduction of the sphere reduces the force experienced by either point charge to $\left(1-24\frac{a^5}{d^5}\right)$ of its initial value.

(d) Consider a toroidal coil of N turns wound uniformly on a form of non-magnetic material with square cross-section of side a. If the mean radius of toroid is R, considering variation of B over the cross-sectional area, show that the self-inductance is given by

$$L = \frac{\mu_0 aN^2}{2\pi} ln \frac{R+\frac{a}{2}}{R-\frac{a}{2}}.$$

(e) If the electrostatic potential at a point (x, y) is given by $V = 2x + 4y$ volts, find the electrostatic energy density. 6+(3+3+6)+10+8+4=40

10. (a) A source of constant voltage V is connect with a capacitor C and a resistor R in series. Obtain an expression of rate of charging in the circuit. Obtain voltage drops across R and C and show their variation with time.

(b) What do you mean r.m.s. value of an ac? Find an expression of it in terms of the peak value of the ac.

(c) Write down Maxwell equations of electromagnetism (in SI units).

(d) State Gibb's phase rule.

Obtain Clausius-Clapeyron's equation in connection with the change of phase due to pressure.

(e) Prove the relation $C_P - C_V = -T\dfrac{\left(\dfrac{\partial V}{\partial T}\right)_P^2}{\left(\dfrac{\partial V}{\partial P}\right)_T}$. (6+3+3)+(3+5)+6+(3+5)+6=40
