2021

MATHEMATICS

PAPER-II

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in ***English*** *or in* ***Bengali*** *but all answers must be in one and same language.*

Group-A

Answer *any five* questions.

1. (a) If $Z_n = \cos\frac{\pi}{2^n} + i\sin\frac{\pi}{2^n}$, Prove that $\lim_{n\to\infty}(Z_1 Z_2 Z_3 \ldots\ldots Z_n) = -1$. 14

   (b) Solve $x^4 + 12x - 5 = 0$ by Ferrarie's Method. 14

2. (a) If a, b, c are three +ve numbers and $a + b + c = 1$, then prove that

   $$\left(a+\frac{1}{a}\right)^2 + \left(b+\frac{1}{b}\right)^2 + \left(c+\frac{1}{c}\right)^2 \geq 33\frac{1}{3}.$$ 14

   (b) If a and b are relatively primes, then prove that gcd $[(a+b),(a-b)] = 2$ or 1. 14

3. (a) Prove that every group of prime order is cyclic. How many generators are there of the cyclic group G of order 8? 10+4=14

   (b) Prove that the intersection of two ideals of R is an ideal of R. 14

4. (a) Prove that the function $f(x,y) = x^3 + 3x^2 + 4xy + y^2$ attains a minimum at the point $\left(\frac{2}{3}, -\frac{4}{3}\right)$. 14

   (b) Verify Green's Theorem in the plane for $\oint_C\{(x^2+xy)dx + xdy\}$, where C is the curve enclosing the region bounded by $y = x^2$ and $y = x$. 14

5. (a) Evaluate, by using suitable transformations $\iint x^2y^2dxdy$ extended over the region $x \geq 0, y \geq 0, x^2 + y^2 \leq 1$. 14

   (b) Let V be the closed region bounded by the surfaces $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$ and $\vec{F} = y\hat{i} + 2x\hat{j} - z\hat{k}$. Find $\iiint_V \vec{\nabla} \times \vec{F}\, dV$. 14

6. (a) Define Complete Metric Space. Prove that every convergent sequence in a Metric Space is a Cauchy sequence.

   The space $C[0,1]$ of all bounded continuous real-valued functions defined in the closed interval [0,1] with the metric d. Prove that $d(f,g) = \max_{0\leq x\leq 1}|f(x) - g(x)|$ is a complete metric space. 2+4+8=14

(b) Show that an analytic function with constant modulus is constant. 14

7. (a) Use Picard's method to compute $y(0 \cdot 2)$ from the differential equation

$\frac{dy}{dx} = 1 + xy$, given $y = 1$ when $x = 0$. 14

(b) The value of Sin x are given below for different values of x. Form a difference table and from this table find Sin 32°. 14

x	:	30°	35°	40°	45°	50°	55°
$y = \sin x$	:	0 · 5000	0 · 5736	0 · 6428	0 · 7071	0 · 7660	0 · 8192

8. (a) Write an algorithm and draw a flowchart to determining a real root of the equation $f(x) = 0$ by Newton-Raphson's method. 4+10=14

(b) A committee consists of the Chairman, President, Secretary and Treasurer. A motion passes if and only if it gets a majority vote or the vote of the Chairman plus one other member. Each member presses a button to indicate approval of a motion. Design a switching circuit controlled by the buttons where the lamp glows if and only if a motion is approved. 14

Group-B

Answer *any two* questions.

9. (a) Let random variable X and Y have joint density function $f(x,y) = \begin{cases} e^{-x-y}, x, y > 0 \\ 0, \text{otherwise} \end{cases}$ and let $U = X + Y, V = \frac{X}{X+Y}$; find the joint distribution function of U and V. 12

(b) A group consists of 3 singers and 4 dancers, who attended a cultural programme. 3 of them at random took dinner in a family. If they were casually requested to sing-a-song, how could they honour their request? Prepare a distribution table. Find the mean and variance from the distribution. 6+6+6=18

10. (a) Obtain the regression coefficients and hence the correlation coefficients from the following data: 15+3=18

Height (in cm)	:	135	140	145	150	155	160	165
Weight (in Kg)	:	46	45	48	50	62	63	65

(b) A normal distribution is given, which has a mean 1·5 and variance 8. If a random sample of size 25 is drawn from that normal population, find the probability that the mean of the sample becomes positive. If again a sample of size 35 is drawn at random, what is the probability that the mean of the sample will be negative? 6+6=12

11. (a) Prove that the set of all feasible solutions to a linear programming problem is a closed convex set. Prove that $X = \left\{X = \frac{(x,y)}{|x|} \leq 2\right\}$ is a convex set. 12+4=16

(b) Find the optimal solution and corresponding cost of transportation in the transportation problem: 14

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	
