2021

PHYSICS

PAPER-I

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer *any three* questions.

1. (a) A particle of mass m travels in a medium along a horizontal linear path. The initial velocity is v_0 and the viscous force acting on it is proportional to its instantaneous velocity. In absence of any other forces, find the velocity of the particle as a function of time.

   (b) Differentiate between inertial and non-inertial frames of reference. A particle is thrown vertically upwards with a velocity v_0 at a place of latitude λ. Show that the particle shifts a distance $\frac{4\omega v_0^3}{3g^2}\cos\lambda$ westward from the original position, where the symbols have their usual meanings.

   (c) Two particles of masses m_1 and m_2 are placed at a distance x apart. Show that at a point where the gravitational field due to two particles is zero, the gravitational potential is given be

   $$V = -\frac{G}{x}(m_1 + m_2 + 2\sqrt{m_1 m_2})$$

   (d) Show that the kinetic energy of a system of particles is equal to the kinetic energy of a single particle of total mass M situated at the centre of mass, together with kinetic energy of the system of particles with their motion relative to the centre of mass.

   (e) A particle moving under a central force describes spiral orbit given by $r = a\exp(b\theta)$, where a and b are constants. Obtain the force law. [6+(4+8)+8+6+8]=40

2. (a) Set up the Lagrangian for the simple pendulum and obtain an equation describing its motion.

   (b) A particle of mass m is dropped into a tunnel drilled along a diameter through a planet of mass M and radius R and of uniform density. Show that the motion of the particle is simple harmonic and also determine its period.

   (c) Show that a shear is equivalent to an equal elongation and a compression at right angles to each other.

   (d) If a number of droplets of water, all of the same radius r cm, coalesce to form a single drop of radius R cm, show that the rise in temperature will be given by $\frac{3S}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$, where S is the surface tension of water and J is Joule's mechanical equivalent of heat.

   (e) Write down the equation of continuity for an ideal fluid of density ρ. Use Poiseuille's formula to show that if two capillary tubes of radii r_1, r_2 and lengths l_1, l_2 respectively are connected in series, the rate of flow V is given by

   $$V = \frac{\pi p}{8\eta}\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)^{-1}$$

   [8+8+8+8+(2+6)]=40

3. (a) Describe briefly Michelson-Morley experiment with a suitable sketch and discuss its significance.

(b) An observer on a railway platform sees two trains approaching each other with speeds v_1 and v_2. He finds that adding v_1 and v_2 gives $\frac{7}{5}c$. An observer on one train sees the other train approaching him with a speed of $\frac{35}{37}c$. What are the velocities of the trains relative to the observer on the platform?

(c) Derive an expression for the relativistic kinetic energy of a body. Show that it reduces to classical value $\frac{1}{2}m_0v^2$ for small velocities($v << c$).

(d) Derive expressions for the moment of inertia of a circular disc of mass M and radius r about (i) an axis perpendicular to its plane, passing through the centre and about (ii) a tangent perpendicular to its plane.

(e) Prove that for a homogeneous and isotropic medium $Y = 3K(1-2\sigma)$, where Y denotes Young's modulus, K Bulk modulus and σ Poisson's ratio. [8+8+(6+2)+(4+4)+8]=40

4. (a) Two sources of intensities $4I$ and I are used in an interference experiment. Obtain the intensities at points where the waves from the two sources superpose with a phase difference of (i) zero and (ii) π. Show that conservation of energy is not violated in the interference pattern.

(b) What is a plane diffraction grating? How many orders would be visible if the wavelength of incident light is 589 *nm* and the number of lines in the grating is 100/*mm*?

(c) Explain Brewster's law. Use this law to show that when light is incident at the polarising angle, the reflected and refracted rays are at right angles.

(d) Show that the ratio

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3}$$

where A's and B's are the Einstein's A and B coefficients and the other symbols have their usual meanings.

(e) A pendulum of length l with a bob of mass m at its end is moving through oil. The bob undergoes small oscillations, but the oil retards the bob's motion with a resistive force $F_{res} = 2m\sqrt{\frac{g}{l}}(l\dot{\theta})$, where θ is the angular displacement of the pendulum. The bob is initially pulled back at $t = 0$ with $\theta = \alpha$ and $\dot{\theta} = 0$. Find the angular displacement θ as a function of time. [(5+5)+(2+4)+(2+4)+10+8]=40

5. (a) Use Gauss's law to calculate electric field intensity due to a spherical charge distribution, given by

$$\rho = \rho_0\left(1-\frac{r}{a}\right), \text{when } r \le a$$
$$= 0, \qquad \text{when } r > a$$

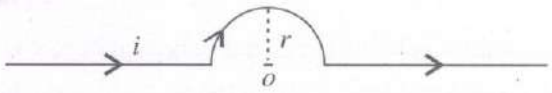
Find also the value of r for which field is maximum.

(b) Two equal charges $+q$ are placed a distance d apart in front of an infinite earthed conducting plane such that each charge is at a distance $\frac{d}{2}$ from the plane. Show that the force on each charge is of magnitude $\frac{3q^2}{8\pi\epsilon_0 d^2}$.

(c) A short electric dipole of dipole moment p_1 is placed in an electric field of another dipole of dipole moment p_2 at a distance d from it. Calculate the force and torque that p_1 experiences when both the dipoles are pointing along the lines joining them.

(d) Apply Kirchhoff's laws to find the current through the galvanometer in an unbalanced Wheatstone bridge. Hence find the condition for balance.

(e) A long wire having a semi-circular loop of radius r carries a current i as shown in the figure. Use Biot-Savart law to find the magnetic induction at the centre O due to the entire wire.



[(5+5)+8+8+(6+2)+6]=40

6. (a) A gas obeys the equation of state $P(v - b) = RT$, where b is a constant. Show that for an adiabatic expansion $P(v - b)^{\gamma}$ = constant, where γ is the ratio of the heat capacities and $C_p - C_v = R$.

(b) Write down Maxwell's relations and use the required relations to show that the internal energy U is a function of T only by finding $\left(\frac{\partial U}{\partial T}\right)_V$, $\left(\frac{\partial U}{\partial V}\right)_T$ and hence $\left(\frac{\partial U}{\partial p}\right)_T$.

(c) 10 g of steam at 100°C are blown on the surface of 90g of water at 0°C, contained in a calorimeter of water equivalent 10 g, all the steam being condensed. Calculate the increase in entropy of the system.

(d) What is Joule-Thomson effect (JT)? Show that for a gas obeying van der Waal's equation of state, the temperature of inversion is approximately $\frac{2a}{bR}$ for the JT effect.

(e) Derive Clausius-Clapeyron equation

$$\frac{dp}{dT} = \frac{L}{T(v_2 - v_1)}$$

from Maxwell's relation. Discuss how the boiling point of a liquid and melting point of a solid are affected by the change of pressure. [8+(4+4)+6+(2+6)+(6+4)]=40

Group-B

Answer *any two* questions.

7. (a) Write down the postulates of special theory of relativity. Using the expression for the relativistic momentum $P = \frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}}$, prove the energy-momentum relation $E^2 = p^2c^2 + m_o^2c^4$, where the symbols have their usual meanings.

(b) Prove that the circle $x^2 + y^2 = a^2$, in a frame S will be seen as an ellipse from another frame S', which is moving with a velocity v along the X-direction with respect to S. Find also the semi-major and semi-minor axis of the ellipse.

(c) Define phase velocity and group velocity. Show that

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda},$$

where v_g is the group velocity, v_p is the phase velocity and λ is the wavelength.

(d) If a vessel has a hole of radius r at its bottom, show that the liquid inside will come out of the vessel if its depth exceeds $h = \frac{2T}{r\rho g}$, where T is the surface tension of the liquid, p its density and g the acceleration due to gravity.

(e) State and prove Bernoulli's theorem for streamline flow of an incompressible non-viscous fluid. [(4+6)+(8+2)+(4+4)+5+7]=40

8. (a) State and explain Fermat's principle. Establish, using the principle, the laws of reflection in a plane surface.

   (b) Derive the following relation

   $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

   for refraction at a convex spherical surface by the matrix method.

   (c) A soap film of refractive index 1.33 is illuminated with light at an angle 45°. There is complete destructive interference for $\lambda = 5890\text{Å}$. Find the thickness of the film. Derive the necessary formula used in the calculation.

   (d) Find the result of superposition of n number of simple harmonic motions having the same amplitude a and the same angular frequency ω, but equal successive phase advancement δ.

   (e) State and explain perpendicular axis theorem. [(4+6)+8+(4+6)+8+4]=40

9. (a) In a series LR circuit with a DC source, find expressions for instantaneous current $I(t)$ in the circuit and instantaneous voltage drop across the inductor V_L. Draw a plot of $I(t), V_L$ versus time and discuss their variations with time.

   (b) Show that the current through pure inductor lags behind the applied ac voltage by a quarter cycle and that through a pure capacitor leads by a quarter cycle.

   (c) Explain how Maxwell generalised Ampere's circuital law. Hence discuss the concept of displacement current.

   (d) Find the state of polarisation when the x-component and y-component of the electric field are $E_x = E_0 \sin(\omega t + kz)$; $E_y = E_0 \cos(\omega t + kz)$.

   (e) Prove that the area of a half-period zone on a plane wavefront is essentially independent of the order of the zone. [(8+2)+(5+5)+(5+3)+6+6]=40

10. (a) Using Laplace's equation find the capacitance of a parallel plate capacitor.

    (b) What do you mean by magnetic scalar potential? Find an expression for the magnetic scalar potential due to a circular current loop at an axial point. Hence calculate the corresponding magnetic field.

    (c) What do you mean by resonance in a series LCR circuit?

    (d) Explain with neat diagram how can you obtain B.H. curve and hysteresis loop of a specimen.

    (e) State and explain what is meant by Gibb's phase rule. Explain why entropy increases in all natural processes. [8+(2+4+4)+4+8+(5+5)]=40
