**2022**

MATHEMATICS

PAPER-II

*Time Allowed — 3 Hours* *Full Marks — 200*

*If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.*

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

**Group-A**

Answer *any five* questions.

1. (a) Find the remainder when $72^{1001}$ is divided by 31.

   (b) Solve $z^8 + z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ in the field of complex numbers. 14+14=28

2. (a) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 - 3x^2 + x - 1 = 0$ from the equation whose roots are $\alpha\beta + \frac{1}{\alpha} - \frac{1}{\beta}$, $\beta\gamma + \frac{1}{\beta} - \frac{1}{\gamma}$, $\alpha\gamma + \frac{1}{\gamma} - \frac{1}{\alpha}$.

   (b) If $\alpha$ be a special root of the equation $x^8 - 1 = 0$, then prove that $(\alpha + 2)(\alpha^2 + 2) \ldots (\alpha^7 + 2) = 85$. 14+14=28

3. (a) Let $G$ be a finite group and $a, b, \in G$. If $b = gag^{-1}$ for some $g \in G$, then prove that $O(a) = O(b)$.

   (b) If $U$ is an ideal of a ring $R$ and let $[R : U] = \{x \in R : rx \in U \ \forall \ r \in R\}$. Prove that $[R : U]$ is an ideal of $R$. 14+14=28

4. (a) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$.

   (b) What is the area of the entire surface formed when the cardioide $r = a(1 + \cos\theta)$ is revolved about the initial line? 14+14=28

5. (a) Find divergence and curl of the vector $\vec{v} = \frac{\hat{r}}{r}$, where $\hat{r}$ is the unit vector along $\vec{r}$ and $r$ is the magnitude of the vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

   (b) Verify Green's theorem in the $xy$–plane for $\oint_C \{(xy + y^2)dx + x^2dy\}$, where $C$ is the closed curve of the region bounded by $y = x$ and $y = x^2$. (7+7)+14=28

6. (a) Let $H$ be the set of all sequence of real numbers $x = \{x_n\}$ such that $|x_n| \leq 1$ for all $n \in N$. Consider the function $d : H \times H \to R$ given by $d(x, y) = \sum_{n \in N} \frac{1}{2^n} |x_n - y_n|$ where $x = \{x_n\}$, $y = \{y_n\} \in H$. Prove that $(H, d)$ is a metric space.

   (b) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of $z$. 14+(5+5+4)=28

7. (a) Evaluate the missing terms in the following table:

| $x$ : | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ : | 0 | — | 8 | 15 | — | 35 |

(b) Find the root of $x^3 - 8x - 4 = 0$, which between 3 and 4, by Newton-Raphson Method, correct to four decimal places. 14+14=28

## Group-B

Answer *any five* questions.

8. Express the Boolean expression in three variables $(x + y + z)\,(xy + x'z)'$ in DNF. 12

9. Draw a flowchart to calculate the mean and standard deviation of $N$ numbers. 12

10. Let $X$ and $Y$ be independent random variables and each be distributed with common mean zero and unit variance. Find the probability density function of $U = \sqrt{X^2 + Y^2}$. 12

11. For a set of bivariate data $x$ and $y$, the lines of regression are $4x + 3y + 7 = 0$ and $2x + 5y = 4$. Identify the lines and hence, find the correlation coefficients between $x$ and $y$. 7+5=12

12. Examine if $X = \{(x_1, x_2)/2x_1 + x_2 \geq 20, x_1 + 2x_2 \leq 80, x_1 + x_2 \leq 50, x_1, x_2 \geq 0\}$ is a convex set. 12

13. Find the optimal solution and the corresponding cost of transportation in the following transportation problem: 10+2=12

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Availability ($a_i$) |
|---|---|---|---|---|---|
| $O_1$ | 19 | 14 | 23 | 11 | 11 |
| $O_2$ | 15 | 16 | 12 | 21 | 13 |
| $O_3$ | 30 | 25 | 16 | 39 | 18 |
| Requirement ($b_j$) | 6 | 10 | 11 | 15 | |

14. Consider the following problem of assigning four operators to four machines. The assignment costs in rupees are given below. Find the optimal cost of assignment. 12

| Operators \ Machines | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 18 | 26 | 17 | 11 |
| 2 | 13 | 28 | 14 | 26 |
| 3 | 38 | 19 | 18 | 15 |
| 4 | 19 | 26 | 24 | 10 |