2022

PHYSICS

PAPER-I

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer *any three* questions.

1. (a) A particle of mass m at rest at $(a, 0, 0)$ is subjected to a force $\vec{F} = -\left(\frac{\alpha}{x^3}\right)\hat{i}$, α is constant. Find the time taken by the particle to reach the origin ($a > 0$, $\alpha > 0$). 10

(b) A body of mass m falls from rest at a height h under gravity through a dense medium of resistive force $F = -kv^2$, where k is constant and v is the speed. Find the kinetic energy when it will hit the ground. 8

(c) Show that if a particle moves in a central force field then its path must be a plane curve. 6

(d) An object at equator is thrown vertically upward with a speed 651·6 km/h. How far from its initial position it will land? 6

(e) A Lagrangian is given by $L = \frac{1}{2}\alpha q^2 - \frac{1}{2}\beta q^2$. α, β constant. Obtain equation of motion. Find the Hamiltonian of the system. 5+5=10

2. (a) Write down the Lorentz transformation equations for a particle moving with velocity $\vec{v}$ along z direction. 4

(b) An event occurs at x = 100 m, y = 10 m, z = 5 m and $t = 10^{-4}$ s in a frame S. Find the coordinates of the event in a frame S' which is moving with velocity $2{\cdot}7 \times 10^8$ m/s with respect to the S along common XX' axis using (i) Gallian transformation, (ii) Lorentz transformation. 4+6=10

(c) Calculate the percentage contraction of a rod moving with a velocity of 0·8c in a direction inclined at 60° to its own length. 8

(d) A particle is rest at origin. A force F starts acting on it at $t = 0$. Show that the speed of the particle at time t is $v = \frac{F.t.c}{\sqrt{m_0^2c^2 + F^2t^2}}$. 8

(e) Determine the S.I. unit of Einstein coefficient A and B. 2+2=4

(f) Determine the ratio of spontaneous to stimulated emission at $T = 10^3 K$ with angular frequency $\omega = 3 \times 10^{15} s^{-1}$. 6

3. (a) State Coulomb's Law and Gauss Law. Among these which Law is more fundamental and why? 3+3+1+3=10

(b) Find the electric field and charge density in a region where the potential is given by $V = a - b\left(x^2 + y^2\right) - c\, ln\left(\sqrt{x^2 + y^2}\right)$ where a, b, c are constant. 4+6=10

(c) A square loop of side a lies in $z = 0$ plane and carries current I. Determine the magnetic field at the center of the loop. 6

(d) Given a magnetic vector potential $\vec{A} = \frac{1}{2}\vec{a} \times \vec{r}$ where a is a constant vector. Find the Magnetic Induction Vector. 6

(e) Two coils of self induction L_1 and L_2 have mutual inductance M. Show from energy consideration that in general $M^2 \leq L_1 L_2$. 8

4. (a) State the Fermat's Principle. Derive the Snell's law using Fermat's Principle. 3+7=10

(b) Two convex lens each of focal length 10 cm and situated 10 cm apart and have common axis an object 2 cm in height is place on the axis at a distance of 15 cm infront of the first lens. Find the position and size of the final image. 6+4=10

(c) Consider a plane convex lens of a material of refractive index 1·5. The convex surface has a radius of curvature 2·5 cm and is face the incident light, center thickness of the lens is 0·6 cm. Construct the System Matrix. 8

(d) What is achromatic doublet? 4

(e) In Young double slit experiment, let β be the fringe width and I_o be the Intensity at the central bright fringe. Calculate the Intensity at a distance x from the central bright fringe. 8

5. (a) For a thermodynamic system the entropy S is related to the internal energy E and volume V by $S = CE^{0.75}V^{0.25}$, C = constant. Find the Gibbs potential. 8

(b) Show that for van der Waals Gas $C_P - C_V = R\left[1 + \frac{2a}{RTV^3}(V - b)\right]$. 8

(c) Define Magnetic Susceptibility of a materials. 4

(d) Calculate the radius of first zone of a zone plate of focal length 0·2 m for a light of wavelength λ = 5000 Å. 4

(e) Red Cadmium line has wavelength 6058Å and Coherence length L = 20 cm. Calculate the line width and Coherence time. 4+4=8

(f) A 40μF capacitor in series with 2 kohm resistor is connected across a 200 volt D.C. source. Determine (i) initial current, (ii) the time constant, (iii) value of the current when time equal to time constant, (iv) energy stored in the capacitor at time 0·04 s. 2+2+2+2=8

6. (a) Define Young Modulus (Y), Bulk Modulus (K) and Rigidity Modulus (n) and show that $Y = \frac{3nK}{n + 3K}$. 3+3+3+8=17

(b) Show that the work done by stretching of a wire is $\frac{1}{2}$ × Young Modulus × (Strain)^2. 7

(c) Two soap bubbles of radius a, b coalesces to form a single bubble of radius c. If the external pressure is B, show that the surface tension is given by $S = \frac{B\left(c^3 - a^3 - b^3\right)}{4\left(a^2 + b^2 - c^2\right)}$. 10

(d) Define coefficient of viscosity. What is the dimension of it? 4+2=6

Group-B

Answer *any two* questions.

7. (a) State the theorems of Parallel and Perpendicular axis as applied to the moment of Inertia. 3+3=6

(b) Find an expression for a moment of Inertia of a solid sphere about a diameter. 10

(c) Obtain an expression for the gravitational self energy of a homogeneous sphere. 8

(d) State the Kepler law of planetary motion. 4

(e) Find the terminal velocity of a oil drop of density 0·95 g/cm^3 and radius 10^{-4} cm fall through the air of density 0·0013 g/cm^3, viscosity of air is 180×10^{-6} C.G.S. and $g = 980\ S^2$. 6

(f) Derive an expression for the equation of continuity of an Ideal fluid of density ρ. 6

8. (a) If a string is plucked at a distance $\frac{l}{4}$ from one end by the amount 'a' then show that the displacement of the string is given by $Y = \sum_1^{\infty} \frac{32a}{3n^2\pi^2} \sin\frac{n\pi}{4} \sin\frac{n\pi x}{l} \cos\frac{n\pi Vt}{l}$. 10

(b) A particle is moving in a straight line with Simple Harmonic motion. Its velocity has the value 5 m/s and 4 m/s when the displacement from the center is 2 m and 3 m respectively. Find the length of the path, frequency of oscillation and phase at a displacement 2 m from the center. 4+4+2=10

(c) Define center of mass of a system of particle. Show that the kinetic energy of a system of particle is equal to the kinetic energy of a single particle of mass M situated at the center of mass together with the kinetic energy of the system of particle with this motion relative to the center of mass. 4+8=12

(d) In Newton ring arrangement of source emitting two wavelength $\lambda_1 = 6 \times 10^{-7}$ m and $\lambda_2 = 5{\cdot}9 \times 10^{-7}$ m. It is found that nth dark ring due to λ_1 coincide with $n + 1$ dark ring of λ_2. Find the diameter of the nth ring if the radius of curvature of the lens is 0·9 m. 8

9. (a) Explain the working principle of He-Neon Laser with energy level diagram. 10

(b) Calculate the thickness of half wave plate for sodium light of wavelength 5893 Å. Given $\mu_0 = 1{\cdot}54$ and ratio of velocity of O-ray and E-ray is 1·007. 6

(c) Calculate the change in entropy when 1 g of ice at –10°C is converted into steam at 100°C. (Specific heat of ice 0·5 cal g^{-1} °C, Latent heat of fusion 80 cal/g, Latent heat of vapour 540 cal/g) 10

(d) What is elastic and inelastic collision? Two particles of mass m_1, m_2 travelling along the same line and collide. Find the velocity after collision for perfectly elastic and inelastic collision. 4+5+5=14

10. (a) What is optical activity? Define specific rotation. 3+3

(b) Describe the state of polarization of light wave represented by $E(z,t)=\hat{i}E_0\sin(kz-wt)-\hat{j}E_0\sin(kz-wt)$. 4

(c) A point charge q of mass m is released from rest at a distance a from an infinite grounded conducting plate. Show that the time taken by the charge to hit the plate is $T=\frac{1}{q}\sqrt{2\pi^3\varepsilon_0 ma^3}$. 10

(d) In one dimentional motion of a mass of 10 gm, is acted upon by a restoring force 10 dyne/cm and a resisting force of 2 dynes–sec/cm.

(i) Find whether the motion is aperiodic or oscillatory.

(ii) Find the value of the resisting force which will make the motion critically damped.

(iii) Find the value of the mass for which the given forces will make the motion critically damped. 4+3+3=10

(e) The vector potential $\vec{A}$ due to a magnetic moment $\vec{m}$ at a point $\vec{r}$ is given by $\vec{A}=\frac{\vec{m}\times\vec{r}}{r^3}$. If $\vec{m}$ is directed along z axis find the x component of magnetic field at the point $\vec{r}$. 10