2022

PHYSICS

PAPER-II

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer *any six* questions.

1. (a) Describe briefly with a diagram the basic principles of Stern-Gerlach experiment.
   (b) In a many-electron atom, the orbital, spin and total angular momentum are denoted by L, S and J. If L = 2, S = 1 and J = 2, find the angle between L and S, using the vector atom model. 7+3=10

2. (a) Prove that $[L^2, L_x] = 0$ where L is the orbital angular momentum and L_x is the x-component of L.
   (b) Using Pauli spin matrix representation, show that the operator
   $$S_x S_y S_z^2 = \frac{\hbar^5}{2^5}\sigma_y .$$
   (c) Prove that $[\sigma_x, \sigma_y] = 2i\sigma_z$. 4+3+3=10

3. (a) The density of copper is 8·96 gm/cc and atomic weight is 63·5 gm. Calculate its Fermi energy. Derive the necessary formula. (Given $h = 6{\cdot}626 \times 10^{-34}$ J-sec)
   (b) Show that the probability that a state ΔE above the Fermi level E_F is filled is equal to the probability that a state ΔE below the Fermi level E_F is empty. (3+4)+3=10

4. (a) A system consists of N particles distributed among four non-degenerate energy states with energies $\in_1 = 0$, $\in_2 = kT$, $\in_3 = 2kT$, $\in_4 = 3kT$, where k is Boltzmann constant and T is absolute temperature. If the total energy of the system is 849 kT, find N.
   (b) What are accessible microstates?
   (c) State Wien's displacement law. Plot the variation of energy density of radiation vs. wavelength for two different temperatures. 5+2+(1+2)=10

5. (a) How does the band theory of solids help to distinguish between metals, insulators and semiconductors?
   (b) Plot the variation of energy, velocity and effective mass of an electron with wave vector "k" according to band theory.
   (c) In a crystalline solid, the energy band structure ($E - k$ relation) for an electron of mass 'm' is given by
   $$E = \frac{\hbar^2 k(2k-3)}{2m}.$$
   Calculate the effective mass of the electron in the crystal. 4+3+3=10

6. (a) Find an expression of Hall coefficient (R_H) in metals.
   (b) Explain analytically why 5-fold rotation axis does not exist in a crystal lattice.
   (c) What is meant by penetration depth in a semiconductor? 4+3+3=10

7. (a) Why does the Coulomb term in semi-emperical binding energy formula
      (i) appear with a negative sign?
      (ii) is directly proportional to square of the atomic number?
      (iii) inversely proportional to one-third power of mass no. (A)?
   (b) Elucidate on the characteristics of the nuclear force. (2+2+2)+4=10

8. (a) Interpret β-decay of neutron in the quark model.
   (b) Find the energy of the photon emitted in the decay $\Sigma^\circ \rightarrow \wedge^\circ + \gamma$.
   Given, mass of $\Sigma^\circ = 1193$ MeV/c^2 and mass of $\wedge^\circ = 1116$ MeV/c^2.
   (c) Which of the following reactions occur?
      (i) $\wedge^\circ \rightarrow \pi^+ + \pi^-$
      (ii) $\pi^- + p \rightarrow n + \pi^\circ$
      (iii) $\pi^+ + p \rightarrow \pi^+ + p + \pi^- + \pi^\circ$ 5+2+3=10

9. (a) Draw the circuit of a $n-p-n$ transistor acting as an amplifier in CE mode. Describe briefly the working of this amplifier.
   (b) A $n-p-n$ transistor in CE mode is used as a simple voltage amplifier with a collector current of 4 mA. The positive terminal of 8 V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B. The collector-emitter voltage $V_{CE} = 4$V, the base-emitter voltage $V_{BE} = 0{\cdot}6$ V and the current amplification factor $\beta = 100$. Calculate the values of R_L and R_B. (2+4)+4=10

Group-B

Answer *any seven* questions.

10. (a) Explain what is meant by 'expectation value' of a dynamical variable in quantum mechanics.
   (b) The wavefunction of a quantum particle of mass m is given by
   $$\psi(x) = A\sin\pi x, \qquad 0 < x \leq 2.$$
   Calculate the value of of $\langle p_x \rangle$ where p_x is the x-component of momentum of the particle.
   (c) Explain the reason for the apperance of a second order space derivative but a first order time derivative in the Schrödinger equation.
   (d) Determine the transmission coefficent for a quantum particle of mass m, energy $E < V_0$ for a ractangular one-dimensional potential barrier of height V_0 defined by
   $V(x) = V_0, \quad 0 < x < a$
   $= 0, \quad x < 0$ and $x > a$. 3+4+3+10=20

11. (a) Defining the raising and lowering operators for a linear harmonic oscillator of mass 'm' and angular frequency 'ω', as
   $$\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}}\left(\hat{p} + im\omega\hat{x}\right)$$
   and $$\hat{a}^+ = \frac{1}{\sqrt{2m\hbar\omega}}\left(\hat{p} - im\omega\hat{x}\right)$$
   obtain the ground state energy and the lowest eigenket.

(b) Show that if the above linear harmonic oscillator is in ground state, then the probability of finding the particle outside the classical limits is approximately 16%.

(c) Prove that the eigenvalues of Hermitian operators are real. 10+4+6=20

12. (a) State Heisenberg's uncertainty principle and determine the radius of the ground state of hydrogen atom.

(b) Consider the wavefunction

$$\psi(r) = A\left(\frac{r}{r_0}\right)e^{ikr}$$

where A is the normalization constant. Calculate the probability current density if $r = 2r_0$.

(c) Show that for an operator $\hat{A}$ corresponding to a dynamical variable A,

$$\langle A^n \rangle = \langle A \rangle^n.$$

(d) The atomic number of sodium is 11.

(i) Write down the electronic configuration for the ground state of the sodium atom.

(ii) Give the standard spectroscopic notation for ground state of sodium and calculate the Lande-g factor. (2+3)+5+5+(2+3)=20

13. (a) Write down the assumptions of Maxwell-Boltzmann statistics. Apply M-B statistics to obtain an expression for partition function of an ideal monoatomic gas in equilibrium at temperature T.

(b) Obtain an expression for mean energy and molar specific heat at constant volume for the ideal monoatomic gas.

(c) A gas consisting of molecules having mass 'm' and obeying MB statistics is in thermal equilibrium at temperature T. The velocity components of the molecules along x–, y– and z-directions are v_x, v_y and v_z respectively. Calculate the mean value of $(v_x + v_y)^2$. 8+7+5=20

14. (a) Mention the drawbacks of Einstein's theory of specific heat of solids.

(b) Show, analytically, how Debye could provide an improved model for explaning the variation of specific heat of solids at constant volume (C_v) with temperature (T).

(c) If the Debye temperature of a salt be 250 K, calculate how much heat is necessary to increase the temperature of one kilomole of the salt from 10 K to 50 K. 3+12+5=20

15. (a) What is a Wigner-Seitz cell?

(b) The primitive basis vectors of a lattice are $\vec{a} = p\left(\hat{i} + 2\hat{j}\right)$, $\vec{b} = p4\hat{j}$, $\vec{c} = p\hat{k}$. Find the primitive translation vectors in the reciprocal lattice space.

(c) Show that the atomic packing factor in a fcc lattice is 0·74.

(d) Show that the reciprocal lattice corresponding to a bcc lattice is a fcc lattice.

(e) Mo-Kα radiation of wavelength 0·71073Å is incident on a simple cubic crystal. The first order Bragg reflection from (131) plane occurs at an angle of 30°. Calculate the lattice parameters of the crystal. 3+4+5+4+4=20

16. (a) What is Larmor precession? Find an expression for its frequency. Hence find an expression for diamagnetic susceptibility using classical theory.

(b) How does the existence of magnetic domains explain ferromagnetism?

(c) Obtain the relation showing the temperature dependence of spontaneous magnetization in a ferromagnetic substance.

(d) If Curie constant is 3×10^{-2} cgs unit and the critical temperature is 600 K, find the value of Weiss molecular field constant. (1+3+4)+3+6+3=20

17. (a) (i) Draw a curve showing average binding energy per nucleon (B/A) of different nuclides as a function of a mass no. A.

(ii) How would you accout for the observed peaks in the region of low values of A in the above curve?

(iii) What can you conclude about the nature of nuclear force from the near constancy of the binding energy per nucleon?

(b) Discuss evidences on the basis of which the nuclear shell model was proposed.

(c) In the reaction

$${}_5B^{11} + {}_2He^4 \rightarrow {}_7N^{14} + {}_0n^1$$

the masses of ${}_5B^{11}$, ${}_7N^{14}$, ${}_2He^4$ nuclei are 11·012 80, 14·00752 and 4·00387 amu respectively. If the incident alpha particle has a kinetic energy of 5·25 MeV towards ${}_5B^{11}$, which is at rest and the kinetic energy of the resultant product nuclei ${}_7N^{14}$ and ${}_0n^1$ are 3·26MeV and 2·139MeV respectively, find the mass of neutron. (4+2+2)+8+4=20

18. (a) Draw the circuit diagram of an integrator using an OPAMP. Obtain an expression of the output voltage.

(b) Design an OP-AMP circuit which gives an output $v_0 = 2v_1 - 3v_2$ where v_1 and v_2 are the two input voltages to the OP-AMP circuit.

(c) Draw energy band diagram for *p-n* junction diode at equilibrium and show how in the case of *p-n* junction diode at equilibrium the potential energy for holes varies with distance from the junction.

(d) The band-gap of a specimen of GaAs is 1·98 eV. Determine the wavelength of the electromagnetic radiation radiated upon direct recombination of holes and electrons in the sample. [Given $h = 6{\cdot}63 \times 10^{-34}$ Joule-sec] (4+4)+5+4+3=20

19. (a) State and explain Barkhausen criteria for sustained oscillation.

(b) Calculate the voltage gain of the negative feedback amplifier with a feedback factor 0·1, if the normal gain of the amplifier is 100.

(c) Sketch the circuit for a NOR gate using diodes and transistors and explain its workings.

(d) Design a logic circuit to implement the following:

$$Y = AB + ABC + \bar{A}B + A\bar{B}C$$

(e) Prove the following Boolean identities:

(i) $A + (B \cdot C) = (A + B) \cdot (A + C)$

(ii) $(A + B) \cdot \overline{(A.\bar{B})} = B$ 3+3+5+3+(3+3)=20