 CSM(O)/STAT-I/22

2022

STATISTICS

PAPER-I

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in ***English*** *or in* ***Bengali*** *but all answers must be in one and same language.*

Group-A

Answer *any four* questions.

1. (a) Give the axiomatic definition of probability. Show that conditional probability satisfies all the axioms.

   (b) Suppose X is distributed with PDF,

   $$f(x)=\frac{1}{2\beta}e^{-|x-\theta|/\beta},\ -\infty<x<\infty.$$

   Show that $P\{|X-\theta|>a+b\,|\,|X-\theta|>a\}$ does not depend on a for any $a, b>0$. 18+12=30

2. Suppose a random variable X has the following PDF,

   $$f(x)=ae^{\left(-x^{2'}-bx\right)},-\infty<x<\infty$$

   for constants a (> 0) and b.

   (a) If $E(X)=-3/2$, find a and b.

   (b) Obtain the second, third and fourth central moments of X. 15+15=30

3. If $(X,Y)\sim N_2(0,0,1,1,\rho)$,

   (a) show that $\text{correlation}\left(X^2,Y^2\right)=\{\text{correlation}(X,Y)\}^2$.

   (b) find $E\left(e^{\frac{1}{2}XY}\right)$. 18+12=30

4. Suppose $X_2\sim N(0,\text{variance}=5)$ and

   $$X_1=1+2X_2-X_2^2/10^3.$$

   (a) Find $\text{Var}(X_1)$ and Cov (X_1, X_2).

   (b) Find an upper bound of $\{\text{Cov}(X_1,X_2)\}^2$ and compare with the actual value. Comment on the results. 12+15+3=30

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5. (a) State and prove Neymar-Fisher Factorization theorem for discrete families of distributions.

   (b) Suppose a N(unknown) faced regular die is thrown 17 times independently. If X_k is the outcome of the kth throw, find a sufficient statistic for N, $k = 1, 2, ..., 17$. 18+12=30

6. (a) Find a most powerful size α test for testing

   $H_0 : X \sim N\,(0, \text{variance} = \tfrac{1}{2})$ ag

   $H_1 : X \sim \text{Cauchy}\,(0, 1)$.

   (b) State and prove Neymar-Pearson Fundamental Lemma. 18+12=30

Group-B

Answer *any two* questions.

7. (a) Define Sampling Frame and Sampling Design with examples.

   (b) Describe the advantages of sample surveys over the census.

   (c) How do you select a simple random sample of 11 households from a list of 112 households in a village without replacement? Describe any two methods. 8+12+20=40

8. (a) Distinguish between stratified random sampling and two stage sampling procedures.

   (b) Under a linear systematic sampling procedure, propose an unbiased estimator of population mean. Also find its variance.

   (c) What is ratio estimator? Find its exact bias. How do you estimate population mean using ratio estimation? Is such an estimator unbiased? 8+12+20=40

9. (a) Describe three basic principles of experimental design.

   (b) What is confounding in the context of factorial designs? Distinguish between partial and complete confounding through examples.

   (c) Consider the one-way ANOVA model

   $$y_{ij} = \mu + \alpha_i + e_{ij},\ i = 1, \cdots, p;\ j = 1, \cdots, n_i$$

   with $\sum_{i=1}^{p} n_i \alpha_i = 0$.

   Define $\text{SSA} = \sum_{i=1}^{p} n_i \left(\bar{y}_{io} - \bar{y}_{oo}\right)^2$

   $$\text{SSE} = \sum_{i,j} \left(y_{ij} - \bar{y}_{io}\right)^2 \text{, where}$$

   $$\bar{y}_{io} = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij},\ \bar{y}_{oo} = \sum_{i=1}^{p} n_i\, \bar{y}_{io} \Big/ \sum_{i=1}^{p} n_i .$$

   If e_{ij}'s are *iid* normal random variables with mean zero and variance σ^2, derive E(SSA) and E(SSE). 12+8+20=40