2023

MATHEMATICS

PAPER-II

Time Allowed — 3 Hours *Full Marks — 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

*Answers may be given either in **English** or in **Bengali** but all answers must be in one and same language.*

Group-A

Answer *any five* questions. (14+14)×5=140

1. (a) Solve the system of linear congruences 14

$$x \equiv 1(\text{mod } 3),$$
$$x \equiv 2(\text{mod } 5),$$
$$x \equiv 3(\text{mod } 7).$$

(b) Using De Moivre's theorem prove that $\sin^4\theta \cos^2\theta = \frac{1}{32}[cos6\theta - 2\cos4\theta - \cos2\theta + 2]$. 14

2. (a) If α, β, γ are the roots of $x^3 + 3x + 2 = 0$, then find (i) $\sum \alpha^5$ (ii) $\sum \alpha\beta(\alpha + \beta)^3$. 7+7

(b) Find the special roots of the equation $x^9 - 1 = 0$ and show that they are roots of the equation $x^6 + x^3 + 1 = 0$. 14

3. (a) If H and K be two finite subgroups of a group (G, .), then show that

$$\circ(HK) = \frac{\circ(H).\ \circ(K)}{\circ(H \cap K)}$$ 14

(b) Let R be a commutative ring with unity, then prove that Ra is a principal ideal of R, generated by $a \in R$. 14

4. (a) Let V be a function of two variables x and y and $x = r\cos\theta$, $y = r\sin\theta$, then prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}.$$ 14

(b) Evaluate $I = \iint_S \left(xdydz + dzdx + xz^2dxdy\right)$, where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 14

5. (a) Show that the vector $\vec{V} = (4xy - z^3)\hat{\imath} + 2x^2\hat{\jmath} - 3xz^2\hat{k}$ is irrotational. Also show that $\vec{V}$ can be expressed as the gradient of some scalar function ϕ. 14

(b) Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 14

6. (a) Let X be the set of all continuous real-valued functions defined on [0, 1], and let $d(x,y) = \int_0^1 |x(t) - y(t)|dt, \forall x, y \in X$, show that (X, d) is not a complete metric space. 14

(b) Let $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

show that $f(z)$ is not analytic at the origin though it satisfies Cauchy – Riemann equations. 14

7. (a) Calculate by Simpson's one-third rule, the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$ correct up to three significant figures, by taking six intervals. 14

(b) Solve the following system of equations by Gauss-Seidel iteration method (up to 3rd approximations). 14

$$20x + 2y + z = 30$$
$$x - 40y + 3z = -75$$
$$2x - y + 10z = 30$$

Group-B

Answer *any five* questions. 12×5=60

8. (a) Write the function $(xy' + xz)' + x'$ in CNF.

(b) Find the complement of $f = (x + y')(x' + y)$. 6+6

9. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ gives the roots of a quadratic equation $ax^2 + bx + c = 0$. Draw a flow chart to solve for real value of x. 12

10. Determine the optimal basic solution to the following transportation problem: 12

	1	2	3	4	a_i
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
b_j	7	5	3	2	

11. Consider the problem of assigning four operators to four machines. The assignment costs in rupees are given here. Operator 1 cannot be assigned to machine III and operator 3 cannot be assigned to machine IV. Find the optimal cost of assignment. 12

		Machines			
		I	II	III	IV
Operators	1	5	5	—	2
	2	7	4	2	3
	3	9	3	5	—
	4	7	2	6	7

12. If the probability density function of a random variable is given by

$f(x) = ce^{-(x^2+2x+3)}$, $-\infty < x < \infty$, find the value of the constant c, the expectation and variance of the distribution. 4+3+5

13. The pdf of a random variable X is assumed to be of the form $f(x) = cx^{\alpha}, 0 \leq x \leq 1$ for some number and constant c. If $X_1, \ldots, X_n$ is a random sample of size n, find the maximum likelihood estimate of α. 12

14. Obtain the basic feasible solutions of the system of equations 12

$$x_1 + 4x_2 - x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 8\,.$$